# Monte Carlo Estimators for Differential Light Transport – Supplemental Material

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#### 1 EXTENDED ESTIMATOR COMPARISON

We return to the common case of computing gradient contributions due to changes in surface roughness. As discussed in the main paper, a multitude of different Monte Carlo estimators can solve this problem. Here we again briefly summarize the individual techniques we can choose and go over the full list of 21 different strategies (5 individual strategies and 16 combinations using MIS). Note that many of them are either incompatible or biased.

### 1.1 Individual strategies

- **Detached emitter sampling**: As we differentiate with respect to the surface roughness, this sampling strategy is in any case independent of  $\pi$  and not part of the differentiation.
- **Detached BSDF sampling**: A standard choice when estimating derivative integrals using the primal sampling technique + PDF.
- *Naïve* attached BSDF sampling: In this strategy, the sampling technique is also part of the differentiation process. It introduces extra discontinuities related to visibility and is thus a biased technique.
- *Reparameterized* attached BSDF sampling: Same as above, but also using our reparameterization approach that freezes the visibility related discontinuities.
- **Differential detached BSDF sampling**: This is a detached estimator, but with a sampling technique specifically designed for the differential BSDF.

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### 1.2 MIS variants

- **Detached weights, detached estimators**: This is the standard variant that applies MIS on top of gradient integrals, i.e. the detached estimators. The weights are not differentiated.
- Attached weights, attached estimators: This is the standard variant that differentiates the complete estimator, including MIS weights and sampling strategies.
- **Detached weights, attached estimators**: This "mixed" strategy only attaches the sampling techniques but detaches the MIS weights. The combined estimator is biased.
- Attached weights, detached estimators: This "mixed" strategy combines two detached estimators, but additionally differentiates the MIS weights as well. We found this method to have negligible differences compared to the (simpler) variant that detaches all components.

#### 1.3 Combination matrix

When now combining emitter sampling and the four BSDF sampling strategies using any of the four MIS variants we arrive, in principle, with an additional 16 estimators. Due to incompatibilities (e.g. the "Attached weights, attached estimators" MIS variant cannot be used together with only detached estimators), only 8 variants are actually realizable, and only 5 of these are actually unbiased.

The following tables shows the complete design space of correct (green), biased (red), and incompatible (gray) combinations. Note that we compare all correct estimators in this document, with exception of the "attached weights, detached estimators" row, which we found to perform equivalently to the version that detaches both weights and estimators (last row). In addition, we also show results using the individual "naïve attached" strategy (without any MIS) to illustrate the discontinuity issue that happens with this strategy.

	Detach
Emitter sampling strategies	correct

	Detach	Reparam. attached	Diff. detached	Naïve attached
BSDF sampling strategies	correct	correct	correct	biased

BSDF sampling variant MIS variant	Detach	Reparam. attached	Diff. detached	Naïve attached
attached weights, attached est.		correct		biased
attached weights, detached est.	correct		correct	
detached weights, attached est.		biased		biased
detached weights, detached est.	correct		correct	

## 1.4 Extended paper figures



Fig. 1. Extended version of the teaser figure including four additional estimators and false color gradient images. Note how the "naïve attached" estimator is missing important gradient contributions that arise from the attached sampling technique and visibility discontinuities in the scene. The last 3 columns show additional results where the three unbiased material sampling techniques are combined with emitter sampling via multiple importance sampling. Like in the primal problem, this always improves robustness.



Fig. 2. Another extended version of the teaser figure, this time visualizing gradients and their standard deviation on the uncropped scene.



Fig. 3. An extended version of the estimator comparison scene where three estimators (involving multiple importance sampling) are added and in addition, false color standard deviation images are shown like for the teaser figure.