

# Simple and Robust Iterative Importance Sampling of Virtual Point Lights

I. Georgiev<sup>†1</sup> and P. Slusallek<sup>1,2‡</sup>

<sup>1</sup>Saarland University, Germany

<sup>2</sup>DFKI Saarbrücken & Intel Visual Computing Institute Saarbrücken, Germany

---

## Abstract

*We present a simple and practical algorithm for importance sampling virtual point lights (VPLs) [Kel97], suitable for multi-pass rendering. During VPL distribution, a Russian roulette decision accepts each VPL proportionally to its estimated contribution to the final image. As a result, more VPLs are concentrated in areas that illuminate the visible parts of the scene, at the cost of a negligible performance overhead in the preprocessing phase. As VPLs are sampled independently and proportionally to their camera importance, the algorithm is trivial to parallelize and remains efficient for low sampling rates. We show that this sampling scheme is well suited to both well illuminated scenes as well as for difficult visibility conditions. Moreover, in contrast to bidirectional and Metropolis VPL sampling [SIMP06, SIP07], the algorithm is fast and very simple to implement, and uses a single Monte Carlo sampler, making it easier to maintain good stratification.*

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Raytracing, Radiosity

---

## 1. Introduction

Efficiently solving the global light transport problem has been a major focus of computer graphics research. Probably the most challenging part of this problem is the simulation of indirect illumination. Instant Radiosity [Kel97] is a robust Monte Carlo method for computing diffuse interreflections that converts indirect illumination into direct illumination by approximating the radiance field in a scene by a set of virtual point lights (VPLs). This set is created by emitting particles from the light sources and propagating them in the scene using random walks, creating a VPL at each hit point.

Instant Radiosity has the intrinsic property that the preprocess – or VPL distribution – phase is much less computationally expensive than the final rendering, as it only involves tracing a small number of paths from the light sources (typically a few dozen to a few thousand). In this short paper we present a practical and easy to implement iterative

VPL sampling algorithm that takes advantage of this property. Its goal is to produce a set of VPLs relevant for the areas seen by the camera at the expense of little preprocess overhead. The algorithm is motivated by the observation that even in scenes with relatively low illumination complexity many VPLs have very little or no contribution to the final image. We introduce a simple extension to the traditional (quasi-) random walk distribution that probabilistically accepts each VPL based to its estimated camera importance. Acceptance probabilities depend only on the average image luminance, hence in a progressive rendering setup the sampling accuracy improves as more information is accumulated from frame to frame. We demonstrate the robustness of the algorithm on scenes with varying visibility settings.

## 2. Background and Related Work

Most of the current state of the art global illumination algorithms are based on Monte Carlo integration (see [DBBS06] for a detailed discussion). While some approaches aim at efficient unbiased rendering relying on aggressive variance reduction, others focus on algorithmic speed and exploiting

---

<sup>†</sup> e-mail: georgiev@cs.uni-saarland.de

<sup>‡</sup> e-mail: slusallek@cs.uni-saarland.de

coherence. Photon Mapping [Jen01] uses random walks to propagate and store light particles, which are then used to estimate the irradiance at a particular point on a surface. Instant Radiosity [Kel97] (IR) uses the same random distribution of particles, which in contrast to Photon Mapping approximate the radiance field in the scene and are therefore called virtual point lights (VPLs).

As any Monte Carlo algorithm, Photon Mapping and Instant Radiosity are prone to high variance. In photon maps variance usually results in low frequency noise in the irradiance estimates, while coarse VPL approximations produce hard shadows in regions with smooth illumination.

## 2.1. Importance-driven Particle Distribution

In order to increase the efficiency of Photon Mapping and Instant Radiosity, a number of techniques try to distribute the particles according to their camera importance.

Peter et al. [PPI98] generate an “importon” map from the camera, which is then used to guide the scattering of photons. Keller et al. [KW00] use the importon map to control the storage of photons.

Visual importance sampling has also been applied for VPLs. Wald et al. [WBS03] estimate the importance of each light source by tracing random paths from the camera before VPL distribution. Bidirectional Instant Radiosity [SIMP06] (BIR) samples VPLs from both the camera and the lights and then resamples them, keeping the most relevant ones for rendering. Metropolis Instant Radiosity [SIP07] (MIR) uses Metropolis-Hastings sampling to generate a set of VPLs that bring the same amount of power to the camera.

While BIR can find relevant VPLs for scenes with difficult visibility, it requires intermediate storage and multiple different sampling strategies, which involves tuning a number of scene-dependent parameters. On the other hand, while MIR is even more efficient for scenes with complex occlusion and requires no importance storage, it is difficult to implement, and maintaining good stratification is not trivial, especially for low sampling rates. In contrast, the sampling scheme proposed in this paper is easy to implement, has only a few intuitive parameters and can generate on the fly a relevant set of VPLs proportionally to their image contribution.

## 3. Probabilistic VPL Acceptance

Our VPL sampling strategy is a simple extension of traditional Instant Radiosity which replaces the deterministic VPL storage on every light path vertex with a Russian Roulette decision. Similarly to [KW00], VPLs are accepted with probability proportional to their camera importance.

As noted by [SIMP06], the best strategy would be to sample VPLs proportionally to their contribution to each integral

(i.e. pixel) individually. Unfortunately, this would defeat the efficiency of exploiting computational coherence. Therefore, we take the whole image as one integral and sample VPLs proportionally to their estimated importance to this one big pixel. In contrast to the Photon Mapping setting [KW00], we can easily compute the importance of each VPL using a direct estimate of its total image contribution.

## 3.1. Formal Derivation

Instant Radiosity approximates the radiance field in the scene using  $N$  VPLs, yielding the following estimate for the irradiance at point  $x$ :

$$E(x) \approx \sum_{i=1}^N G_i(x) V_i(x) L_i. \quad (1)$$

Here,  $L_i$  is the intensity of VPL  $i$ , and  $G_i$  and  $V_i$  are respectively the geometric and visibility terms between the point and the VPL. In the spirit of [KW00], we transform Equation 1, introducing the acceptance probability  $p_i$  for VPL  $i$ :

$$\begin{aligned} E(x) &\approx \sum_{i=1}^N G_i(x) V_i(x) \frac{L_i}{p_i} p_i \\ &= \sum_{i=1}^N G_i(x) V_i(x) \frac{L_i}{p_i} \int_0^1 \chi_{[0, p_i]}(t) dt, \end{aligned} \quad (2)$$

where  $\chi$  is the characteristic function of the interval  $[0, p_i]$ . Similarly to Russian Roulette absorption [AK], one-sample Monte Carlo estimation of the integral yields

$$p_i = \int_0^1 \chi_{[0, p_i]}(t) dt \approx \chi_{[0, p_i]}(\xi) = \begin{cases} 1 & \text{if } \xi \leq p_i \\ 0 & \text{else,} \end{cases}$$

where  $\xi \in [0, 1)$  is a uniform (quasi-)random variable. This means that instead of being deterministic as in traditional Instant Radiosity, the acceptance of VPL  $i$  now depends on an unbiased random decision based on  $p_i$ . In essence, if  $p_i = 0$  the VPL is simply discarded, as its contribution will be zero. Otherwise, it is stored with intensity scaled by  $1/p_i$  which ensures energy conservation. In the next section we describe how we choose the acceptance probability  $p_i$  in a way that allows us to control the density of VPLs according to their camera importance.

## 3.2. The Acceptance Probability

Given the one-pixel image assumption, our goal is to sample VPLs proportionally to the total power they transfer to the camera. This means that on average each VPL should contribute roughly the same luminance to the whole image.

Suppose we have  $P$  pixels in the image and want to sample  $N$  VPLs. If  $\Phi^{total}$  is the total VPL illumination luminance in the final image and  $\Phi = \Phi^{total}/P$  is its pixel average, our goal is to sample a set of VPLs each contributing total power  $\Phi_v^{total} = \Phi^{total}/N$  and  $\Phi_v = \Phi/N$  per pixel on average.

Assume we have an estimate  $\tilde{\Phi}_v$  of the desired average VPL pixel contribution  $\Phi_v$  and an estimate  $\tilde{\Phi}_i$  of the average pixel contribution  $\Phi_i$  of each VPL candidate. We define the following acceptance probability:

$$p_i = \min\left(\frac{\tilde{\Phi}_i}{\tilde{\Phi}_v} + \varepsilon, 1\right), \quad (3)$$

where  $\varepsilon > 0$  is a small offset ensuring a non-zero value for  $p_i$ . Using the so-defined acceptance probability, VPLs with an estimated contribution higher than the desired average will be trivially accepted with unmodified energy, thus avoiding additional variance. In contrast, VPLs with low relative contribution are likely to be rejected. As a result, more VPLs will be concentrated in the areas around the camera, illuminating the visible parts of the scene.

The parameter  $\varepsilon$  in Equation 3 can be viewed as a trade-off between computational and sampling efficiency. If  $\varepsilon \geq 1$ , the resulting VPL set will be identical to the one created by traditional IR, as all VPLs will be trivially accepted.

### 3.3. Implementation

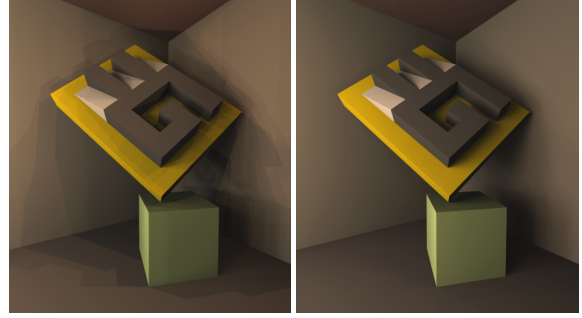
In this section we describe how we compute  $\tilde{\Phi}_i$  and  $\tilde{\Phi}^{total}$ , required for the acceptance probability  $p_i$ . From Section 3.2 it follows that  $\tilde{\Phi}_v = \tilde{\Phi}^{total} / PN$ , where  $\tilde{\Phi}^{total}$  is an estimate of the total VPL illumination luminance.

In order to obtain pixel contribution estimates for the VPL candidates, we trace a small number of rays from the camera at the beginning of each frame and store the hit points. This is the only importance storage required by the algorithm. During VPL sampling, each VPL is connected to these hit points to estimate its average pixel contribution  $\tilde{\Phi}_i$ .

The described scheme for estimating the VPL image contribution can be also viewed as transferring camera importance to the VPLs. Therefore, the camera samples are the equivalent of importons from the Photon Mapping setting.

The other component of the VPL acceptance probability is the total VPL illumination luminance estimate  $\tilde{\Phi}^{total}$ . In a progressive rendering scenario we can use the rendered image from one frame to compute  $\tilde{\Phi}^{total}$  for the next one. When accumulating multiple images, at every iteration we obtain more accurate  $p_i$ 's as  $\tilde{\Phi}^{total}$  converges to  $\Phi^{total}$ . For the first frame  $\tilde{\Phi}^{total}$  is initialized to zero. This results in no importance sampling for the initial VPL set, as  $p_i$  will be clamped to 1 for each VPL. Over time, acceptance probabilities become less conservative, and the uniform exploration of the VPL space gradually moves towards more aggressive importance sampling.

In case only a single image needs to be rendered, VPL sampling and rendering can be performed in multiple passes using the scheme described above. Alternatively,  $\tilde{\Phi}^{total}$  can be computed either using a number of camera samples and VPLs or with (bidirectional) path tracing, and the final VPL sampling and rendering can be performed in one pass.



**Figure 1:** The EG scene rendered with the same amount of VPLs by standard VPL sampling (left) and our importance sampling algorithm (right).

## 4. Results and Discussion

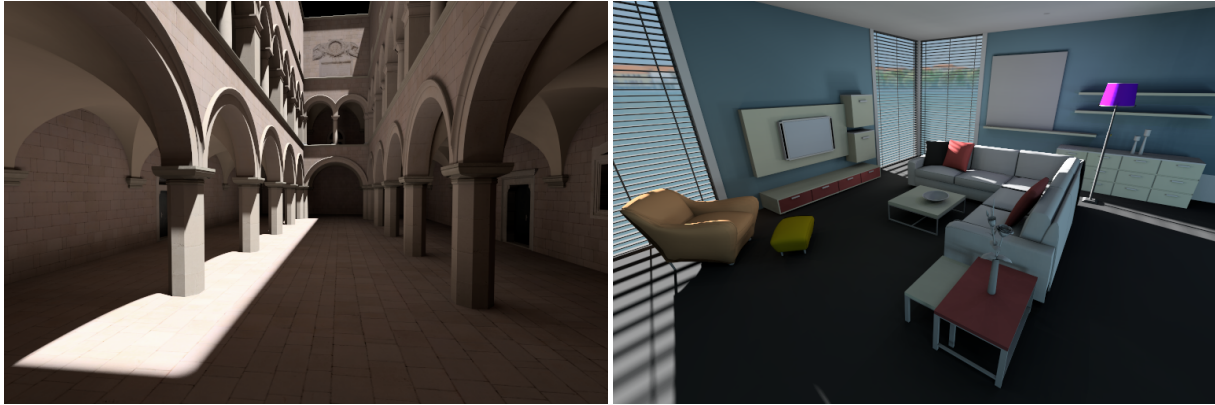
We have implemented the VPL sampling algorithm described in Section 3 in an interactive rendering system, where the camera and the objects can be manipulated in real time, while a high quality image is progressively obtained when interaction is stopped. We handle direct illumination separately, as it usually has quite different frequency characteristics than indirect illumination, thus requiring different sampling schemes. Therefore, the statistical quantities used for importance sampling only account for indirect illumination due to VPLs. We show results for three scenes with varying illumination and occlusion characteristics.

Our tests show that for the same light paths our VPL importance sampling algorithm results in a very similar image to classic Instant Radiosity, but using a fraction of the VPLs. This fraction is equal to the average VPL acceptance probability. For the tested scenes, the overhead of generating more VPL candidates, due to the Russian Roulette acceptance, did not produce noticeable increase in the total rendering times.

SPONZA is a common example scene for one- and two-bounce indirect illumination. The variant shown in Figure 2 left has one directional light source, and the image was rendered with an average VPL acceptance probability of 0.28.

LIVING ROOM (Figure 2 right) is a typical architectural scene, where light can only reach the interior through the blinds on the windows. Almost all illumination in the scene is indirect. Traditional VPL sampling had difficulties in finding the relevant paths through the blinds, and our VPL importance sampling resulted in an average acceptance probability of 0.23.

The EG scene (Figure 1) has one point light source placed around the corner behind the camera. It takes at least two bounces for light to reach the areas visible through the camera. On this scene our importance sampling algorithm discarded 93% of the VPLs (with average  $p_i = 0.07$ ), resulting in more than an order of magnitude higher rendering efficiency than traditional VPL sampling.



**Figure 2:** Left: The SPONZA scene rendered with about 4 times fewer VPLs than when using standard VPL sampling. Right: The LIVING ROOM scene. 77% of the VPL candidates get discarded by our algorithm.

An intrinsic feature of our algorithm is that VPLs are sampled on the fly, independently, and with the need for very little importance storage, namely the camera samples. This allows for easy parallelization and also coupling of VPL sampling and rendering, where different threads only need different sampling seeds.

As we introduce one simple additional sampling step, the VPL stratification yielded by the random walk is preserved in well illuminated visible parts of the scene. The probabilistic storage controls the VPL density in areas low contribution to the final image. In cases of numerical problems where the VPL acceptance probability is very low, the algorithm can be slightly biased to always discard such VPLs. Alternatively, the acceptance probability can be bound from below with a proper value for  $\epsilon$  (see Equation 3), which does not void the unbiasedness of the sampling.

Our algorithm has only a couple of parameters, namely the acceptance probability  $\epsilon$  and the number of camera samples  $N_C$ . In our tests we did not experience the need for tweaking them individually for the different scenes, and used  $\epsilon = 0.05$  and  $N_C = 100$ . Only for the case where a single image needs to be re-rendered and the average image VPL illumination luminance is computed independently, parameters may need to be adjusted depending on the view point and the algorithm used.

## 5. Conclusion and Future Work

In this short paper we presented a simple VPL importance sampling algorithm, which requires almost no importance storage, has few intuitive parameters, and is very easy to implement. The algorithm automatically adapts the sampling probabilities as images are accumulated in a multi-pass setup. As a result, it can robustly find relevant VPLs in various illumination and visibility conditions, even though VPLs are distributed only from the light sources. We believe that

the algorithm will be a useful addition to any Instant Radiosity based renderer mainly because it is very simple and easy to control and parallelize.

One useful extension of any importance-driven VPL sampling algorithm, including ours, is relaxing the assumption of a one-pixel image. Statistics can be gathered for different view point regions, which will possibly have different relevant VPL subsets. This will result in additional efficiency improvement for most scenes.

## References

- [AK] ARVO J., KIRK D.: Particle Transport and Image Synthesis. In *SIGGRAPH '90: Proceedings of the 17th annual conference on Computer graphics and interactive techniques*. 2
- [DBBS06] DUTRE P., BALA K., BEKAERT P., SHIRLEY P.: *Advanced Global Illumination, 2nd edition*. 2006. 1
- [Jen01] JENSEN H. W.: *Realistic Image Synthesis using Photon Mapping*. A.K. Peters, 2001. 2
- [Kel97] KELLER A.: Instant Radiosity. *SIGGRAPH '97 (Proceedings of the 24th annual conference on Computer graphics and interactive techniques)* (1997). 1, 2
- [KW00] KELLER A., WALD I.: Efficient Importance Sampling Techniques for the Photon Map. In *Vision, Modeling, and Visualization* (2000). 2
- [PPI98] PETER I., PIETREK G., INFORMATIK F.: Importance Driven Construction of Photon Maps. In *In Rendering Techniques '98 (Proceedings of the 9th Eurographics Workshop on Rendering)* (1998). 2
- [SIMP06] SEGOVIA B., IEHL J.-C., MITANCHEY R., PÉROCHE B.: Bidirectional Instant Radiosity. In *Proceedings of the 17th Eurographics Workshop on Rendering* (2006). 1, 2
- [SIP07] SEGOVIA B., IEHL J.-C., PÉROCHE B.: Metropolis Instant Radiosity. In *Proceedings of Eurographics 2007* (2007). 1, 2
- [WBS03] WALD I., BENTHIN C., SLUSALLEK P.: Interactive Global Illumination in Complex and Highly Occluded Environments. In *EGRW '03: Proceedings of the 14th Eurographics workshop on Rendering* (2003). 2