Integral formulations of volumetric transmittance

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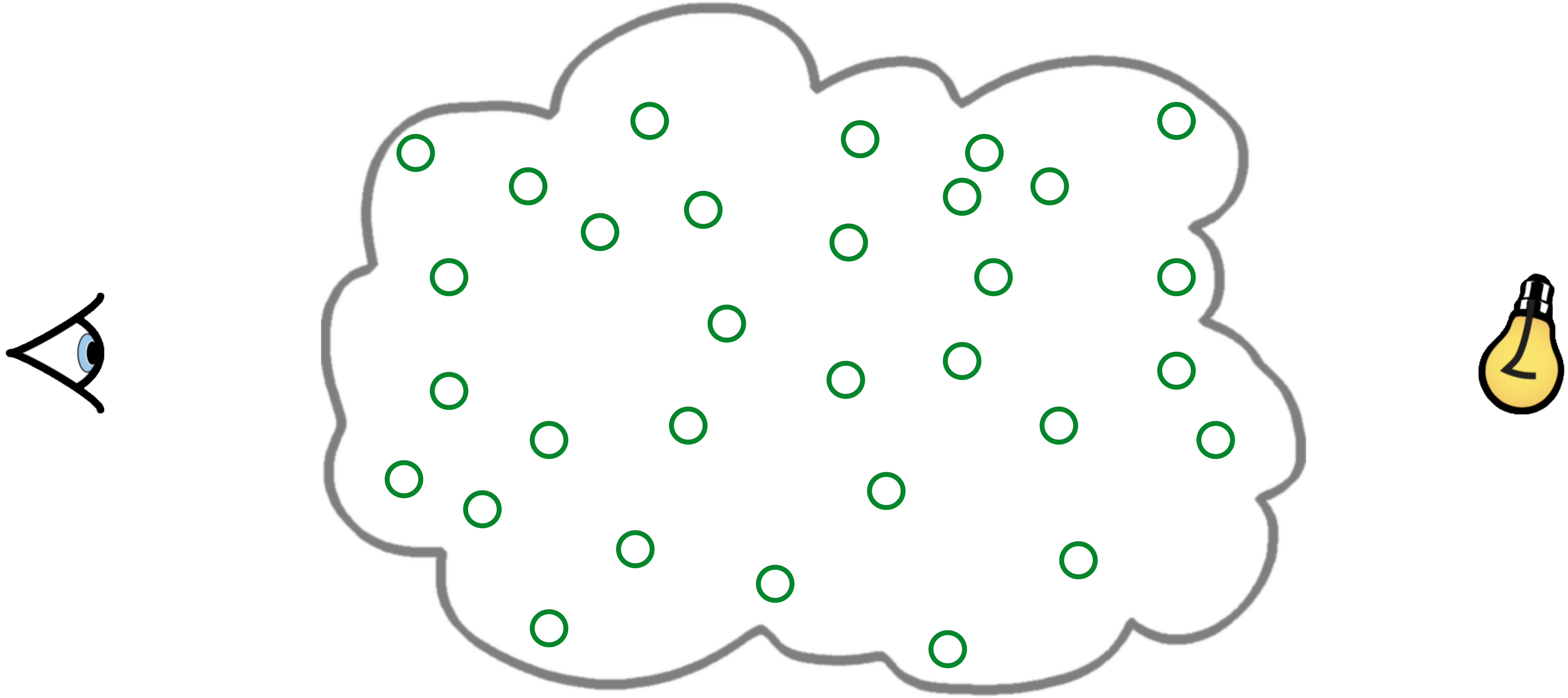
Rendering volumetrics

Rendering volumetrics



Rendering volumetrics

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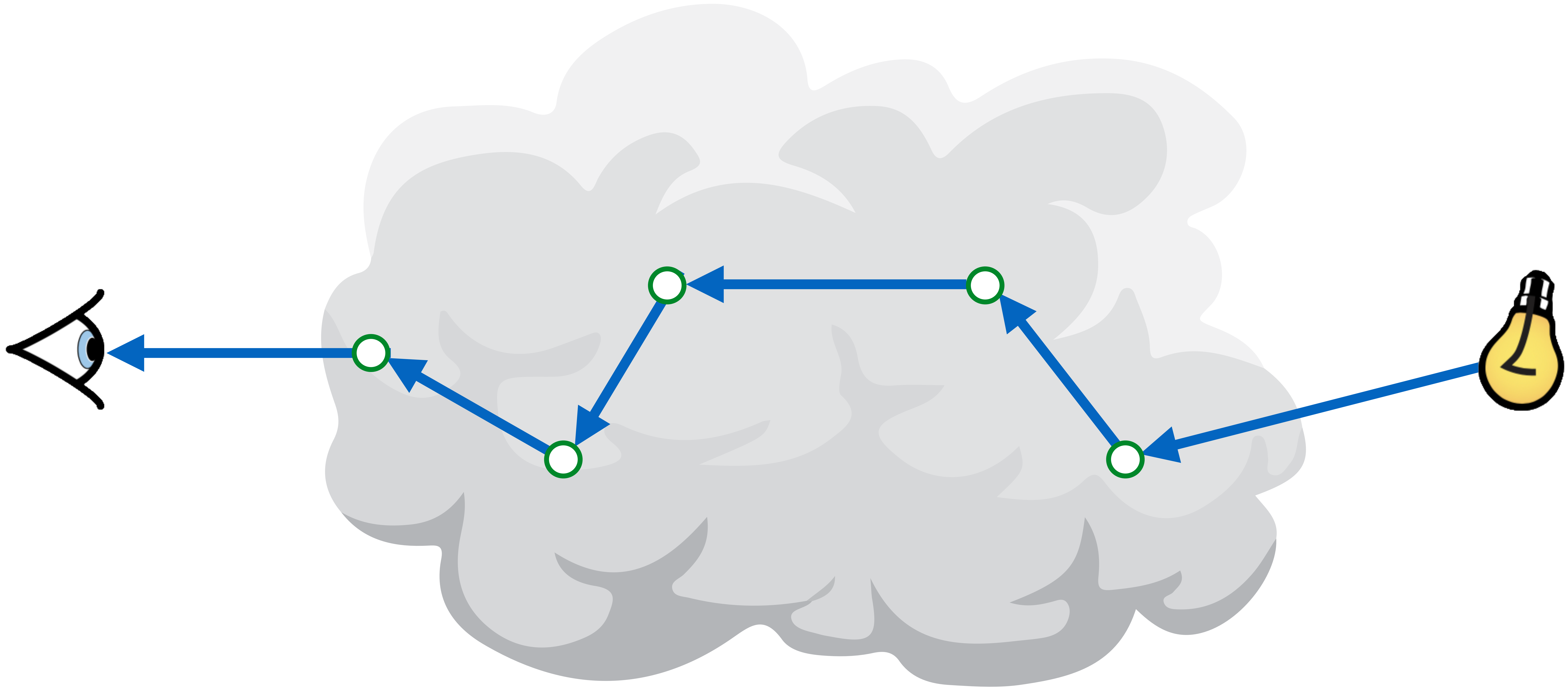
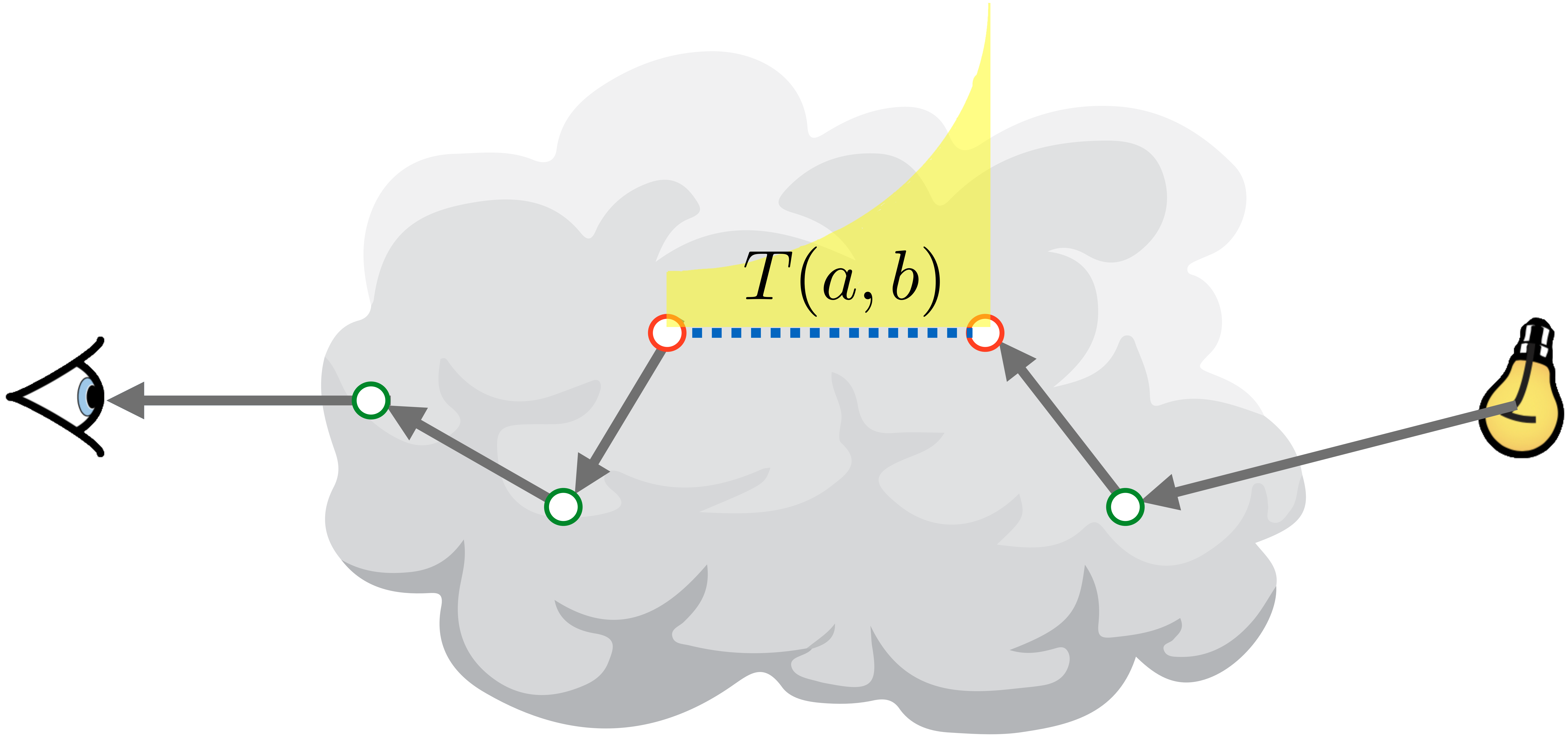


Rendering volumetrics



Transmittance

$$T(a,b) = \frac{L(a)}{L(b)}$$

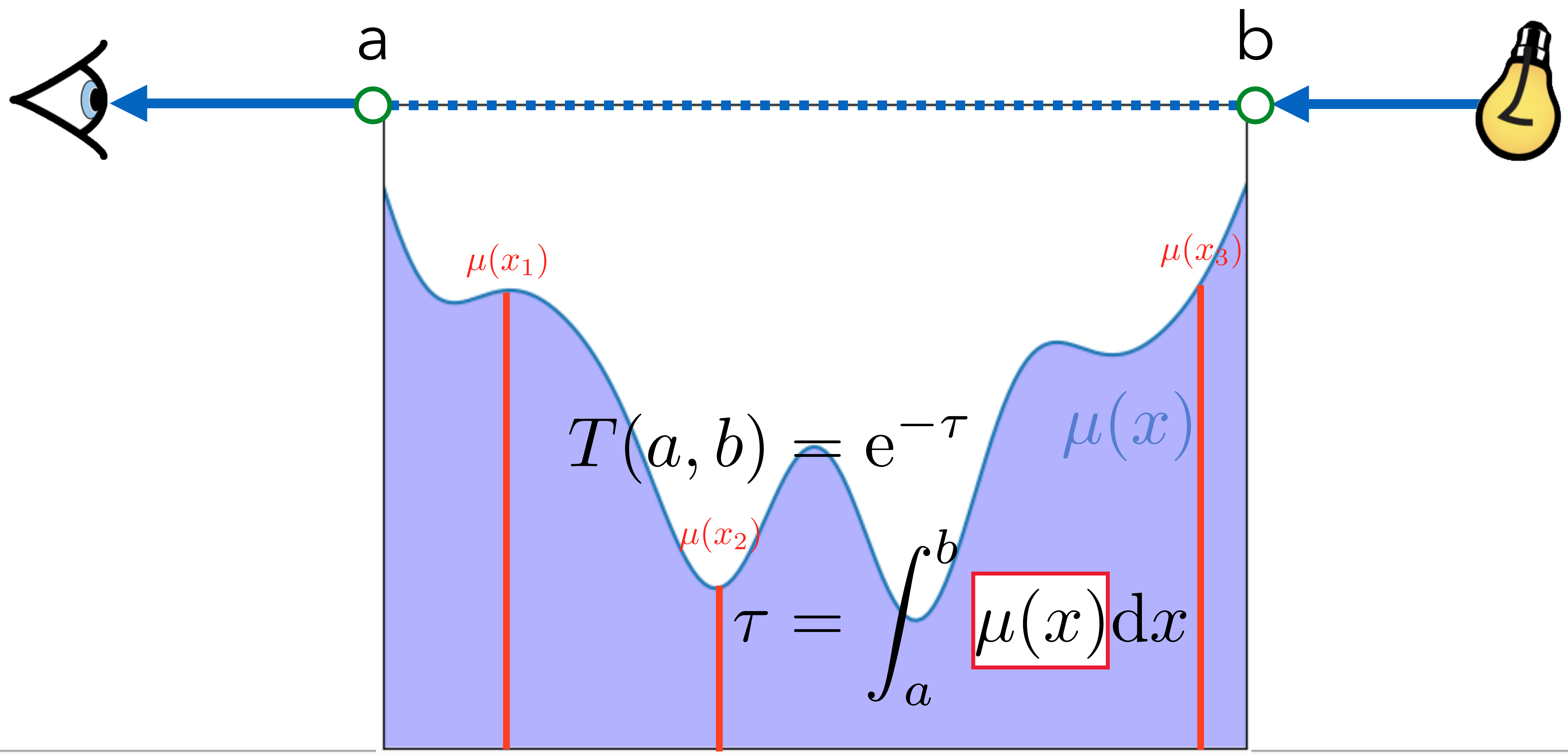
Transmittance

$$T(a,b) = \frac{L(a)}{L(b)}$$

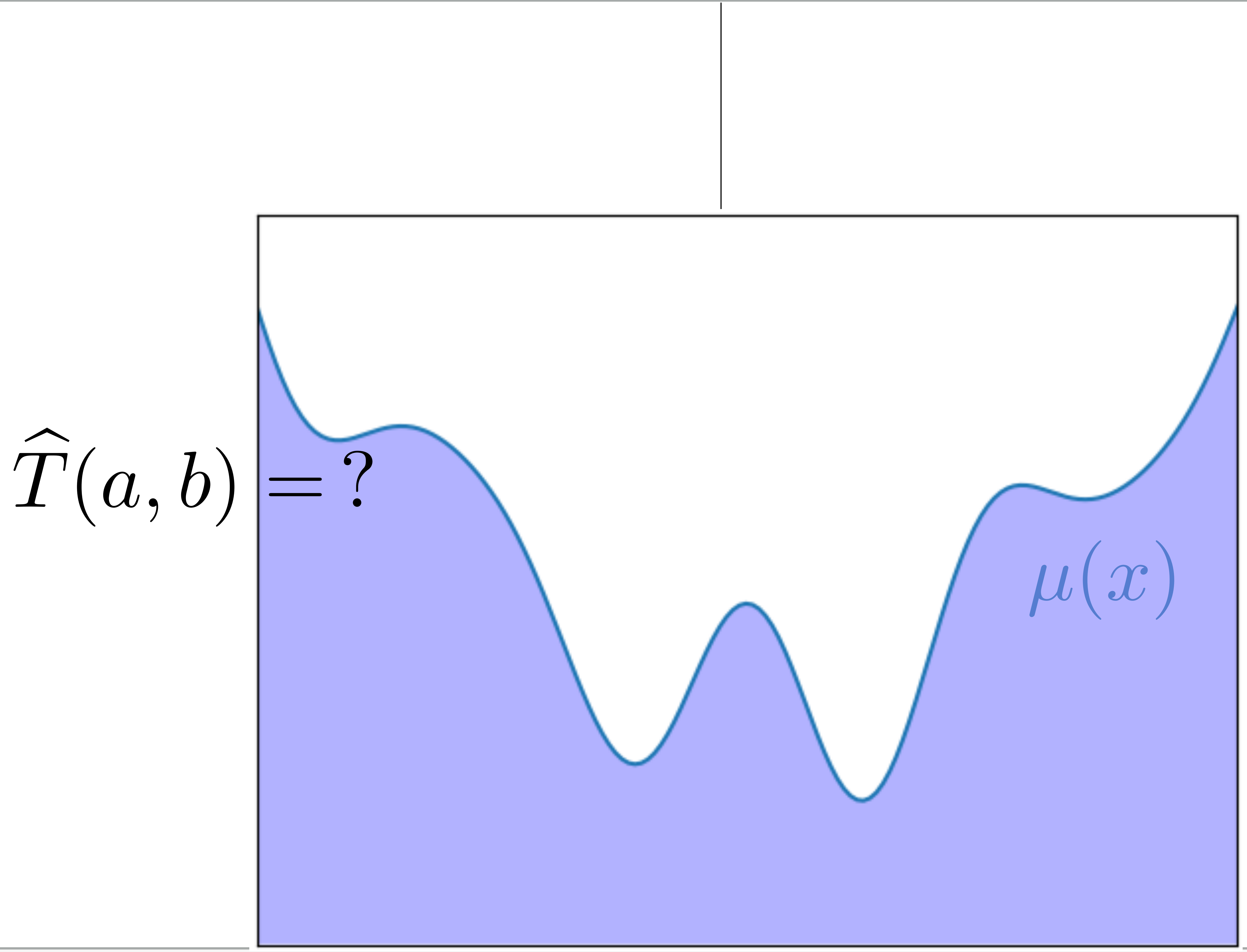
$$T(a,b) = \mathrm{e}^{-\boxed{\tau}}$$

$$\tau = \int_a^b \boxed{\mu(x)}\,\mathrm{d}x$$

Extinction



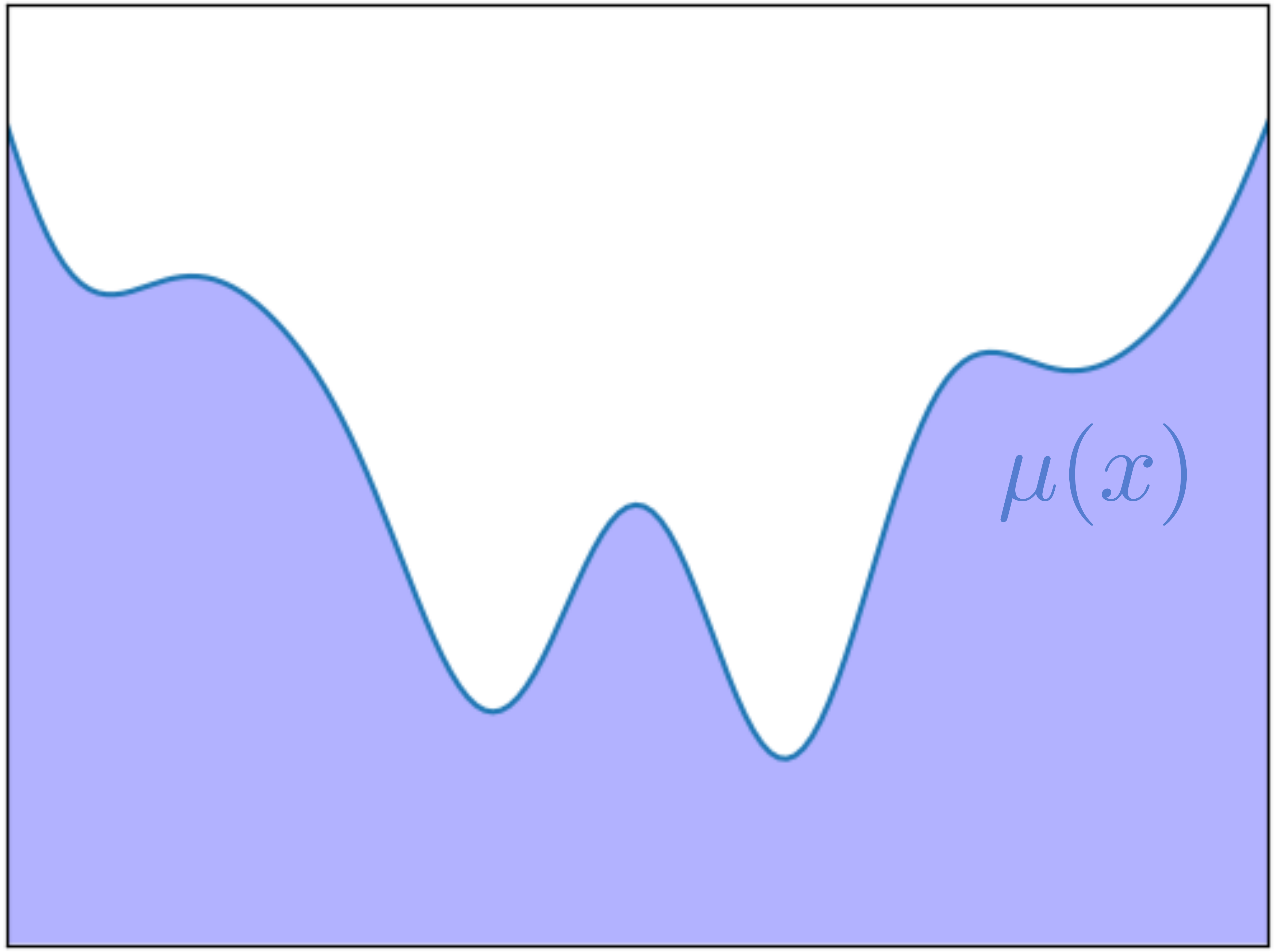
Previous work



Previous work

Delta tracking*

$$\widehat{T}(a,b) = ?$$

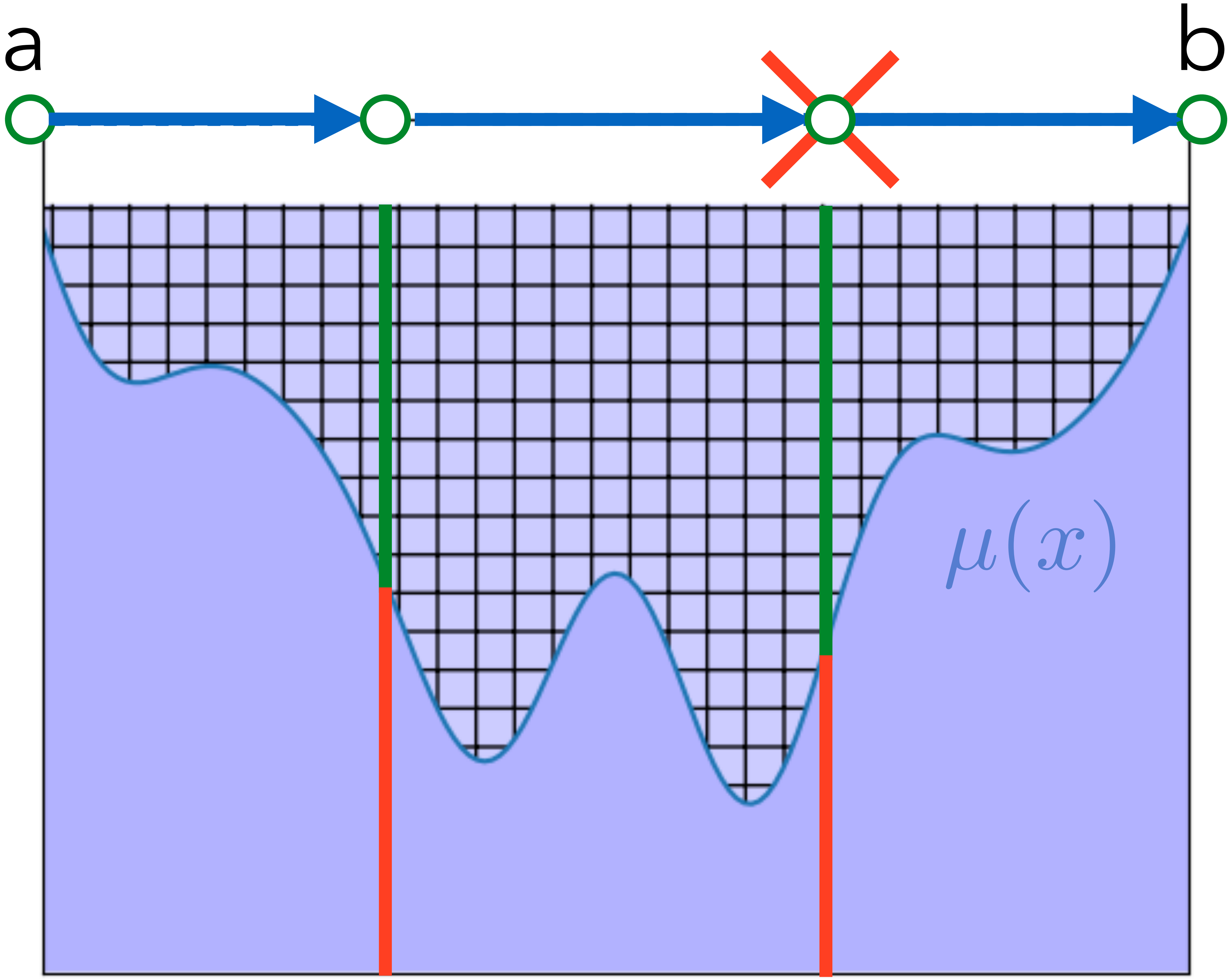


*[Woodcock et al. 1965]

Previous work

Delta tracking*

$$\widehat{T}(a,b) = ?$$



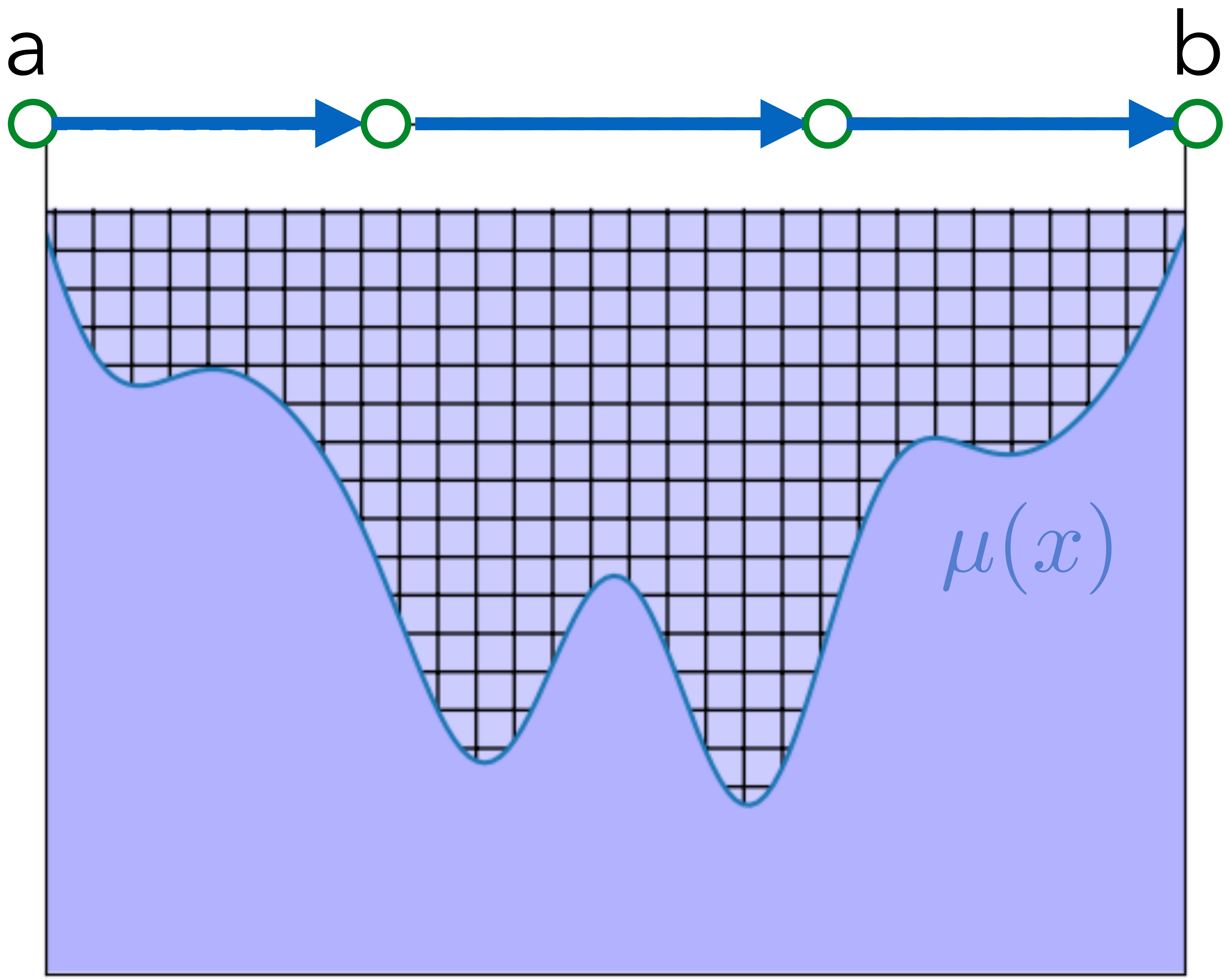
*[Woodcock et al. 1965]

Previous work

Delta tracking*

$$\widehat{T}(a,b) = ?$$

$$E[\widehat{T}_{DT}(a,b)] = \mathrm{e}^{-\tau}$$



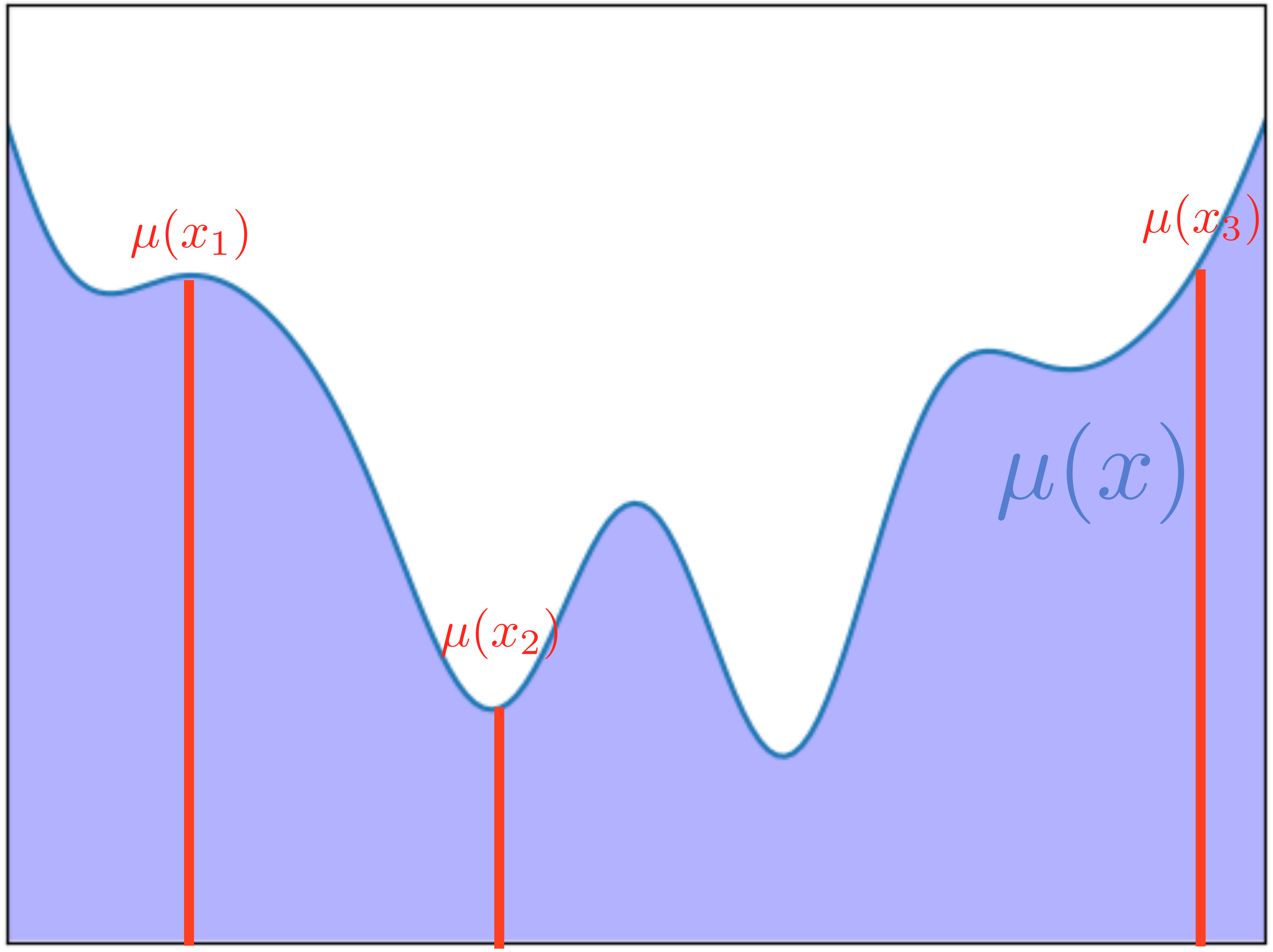
*[Woodcock et al. 1965]

Naive Monte Carlo

$$E[\mathrm{e}^{-\tau}] \neq \mathrm{e}^{-E[\widehat{\tau}]}$$

$$\tau = \int_a^b \mu(x)\mathrm{d}x$$

$$\widehat{\tau} = \sum_{j=0}^{k} \frac{\mu(x_j)}{p(x_j)}$$



Our contributions

$$T(a,b) = \mathrm{e}^{-\tau}$$

$$E[\mathrm{e}^{-\tau}] \neq \mathrm{e}^{-E[\widehat{\tau}]}$$

$$T(a,b) = \int \dots$$

- 3 integral formulations
- Rendering analogies
- MC design decisions

Deriving an integral formulation

$$-\frac{\mathrm{d}L(x)}{\mathrm{d}x} = -\mu(x)L(x)$$

$$\int_a^b -\frac{\mathrm{d}L(x)}{\mathrm{d}x}\mathrm{d}x = \int_a^b -\mu(x)L(x)\mathrm{d}x$$

$$L(a) - L(b) = \int_a^b -\mu(x)L(x)\mathrm{d}x$$

$$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$$

First integral formulation

$$T(a,b) = e^{-\boxed{\int_a^b \mu(x)\mathrm{d}x}}$$

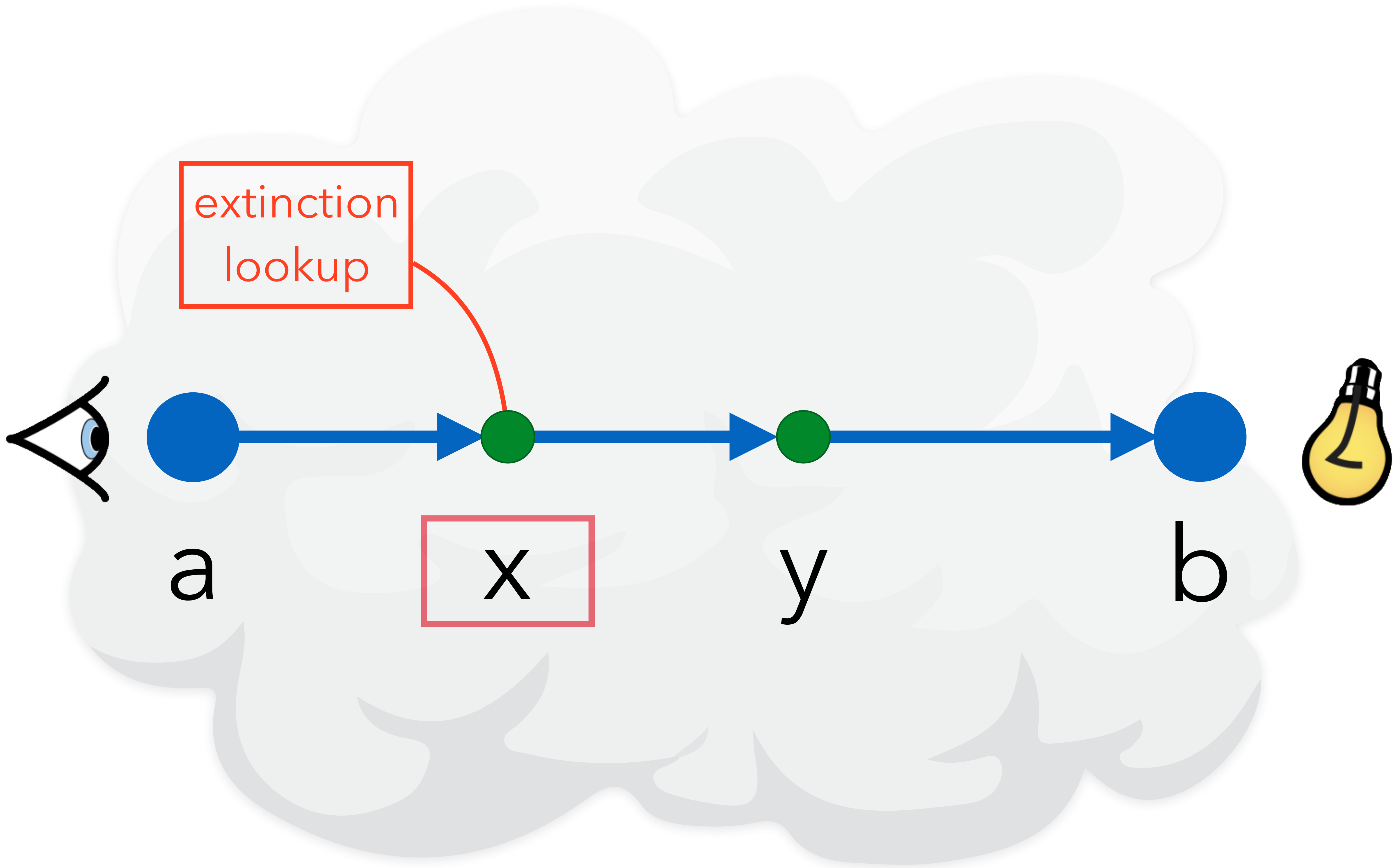
$$T(a,b) = 1 + \boxed{\int_a^b -\mu(x)\boxed{T(x,b)}\mathrm{d}x}$$

$$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$$

Visualizing an estimator

$$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$$

Visualizing an estimator

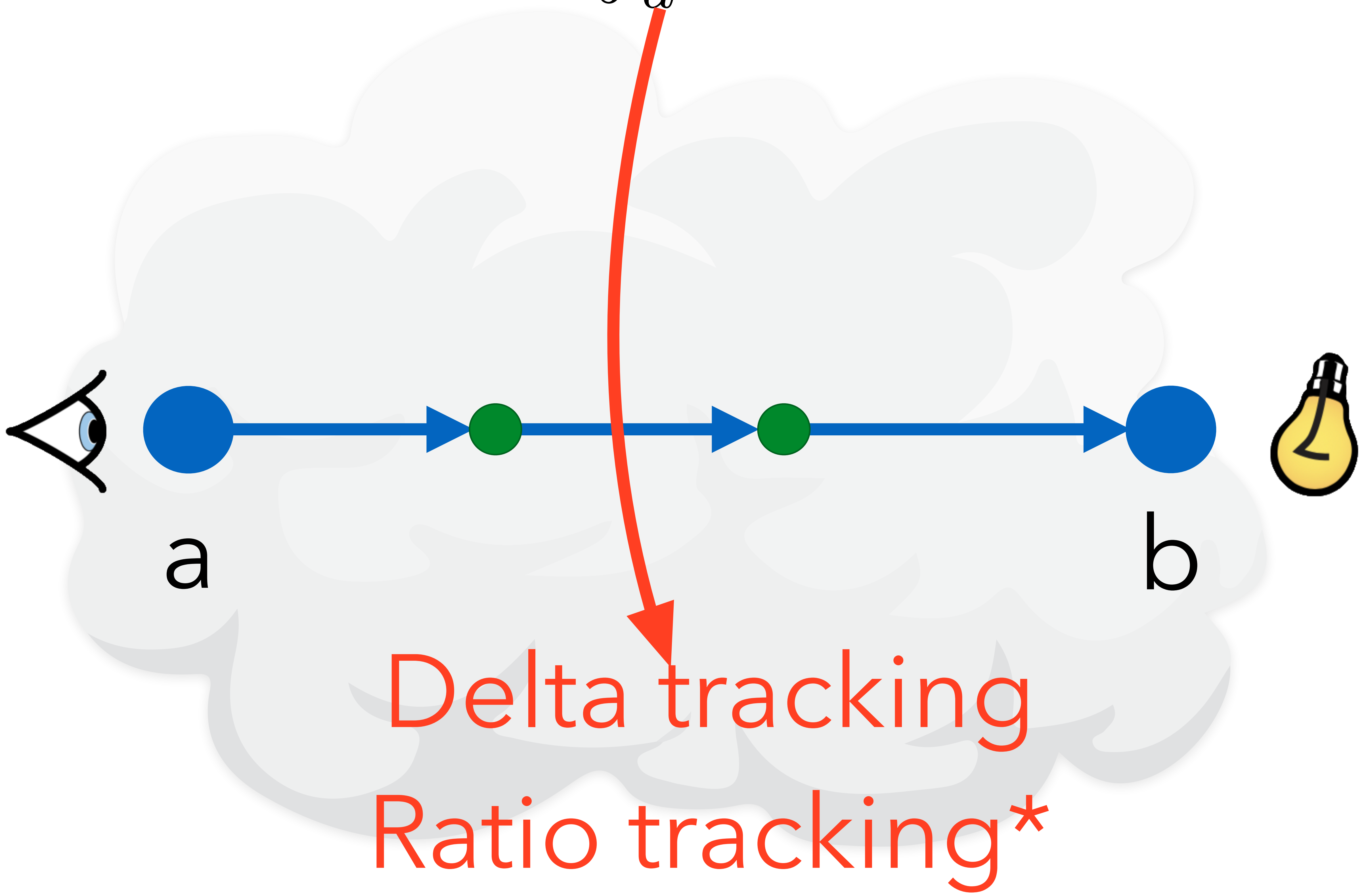


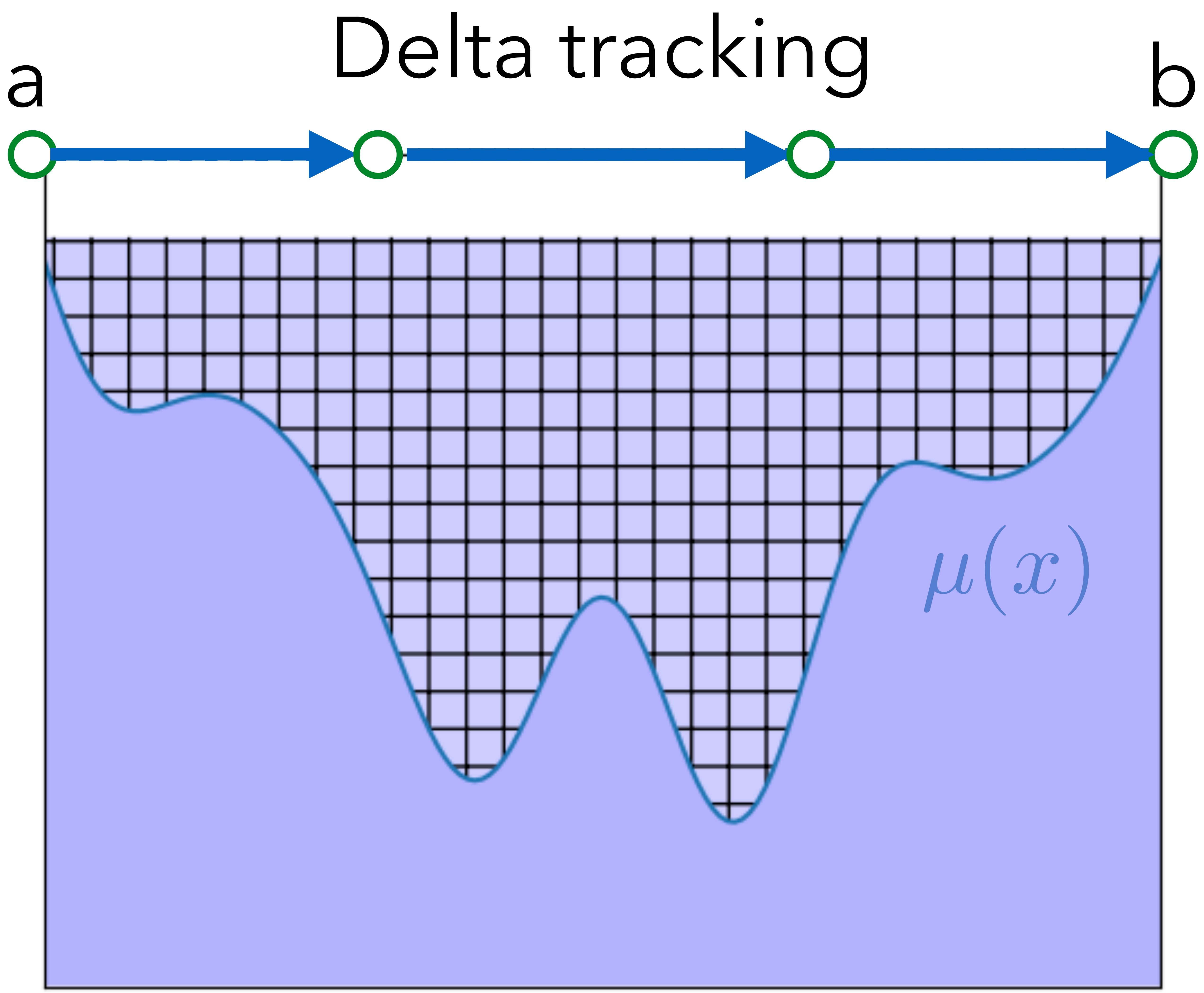
$$\widehat{T}(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\,\mathrm{d}x$$

$$\widehat{T}(a,b) = 1 - \frac{\mu(y)\widehat{T}(y,b)}{p(y)}$$

Analogy to previous work

$$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$$



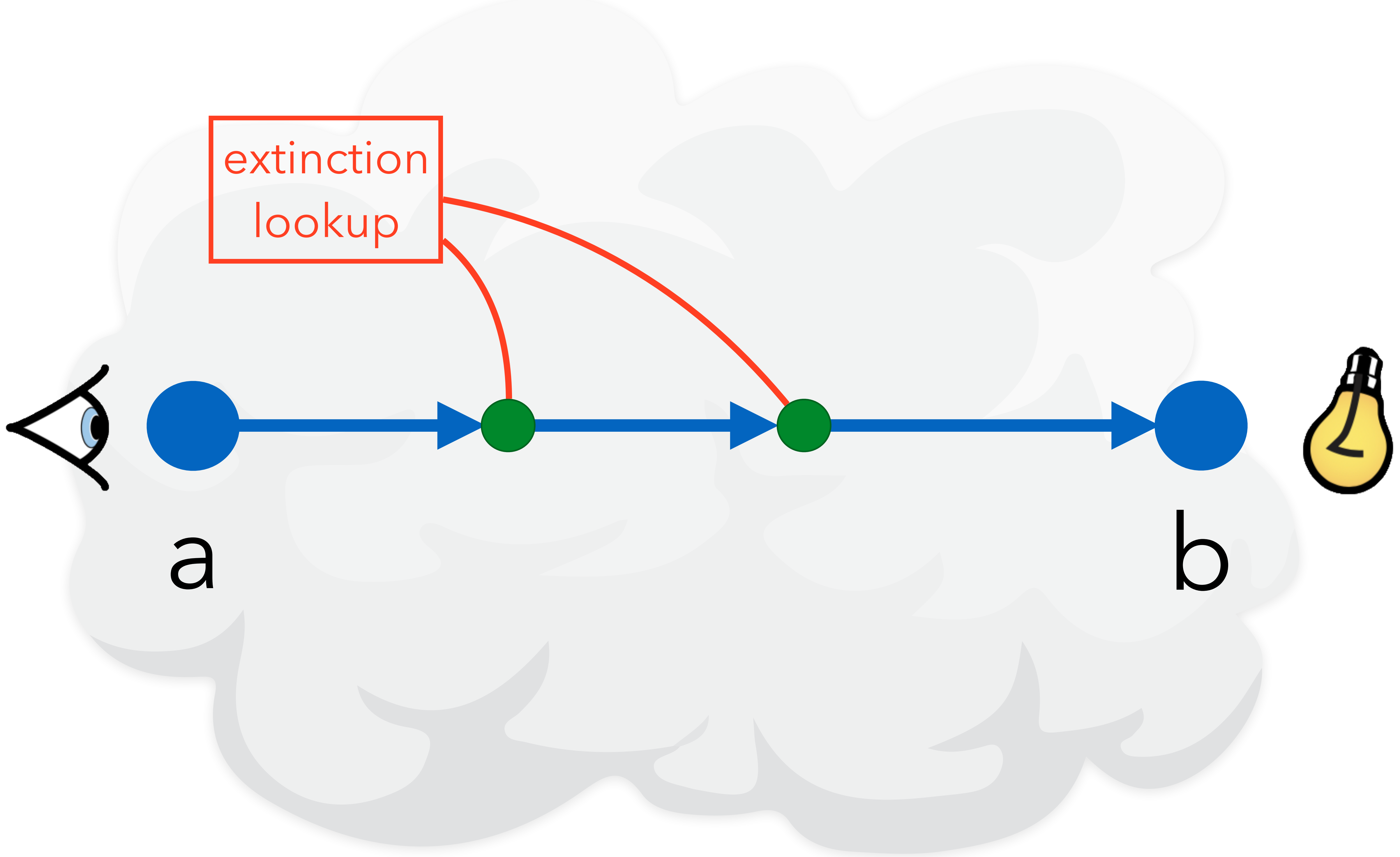


*[Novák et al. 2014]

Analogy to rendering

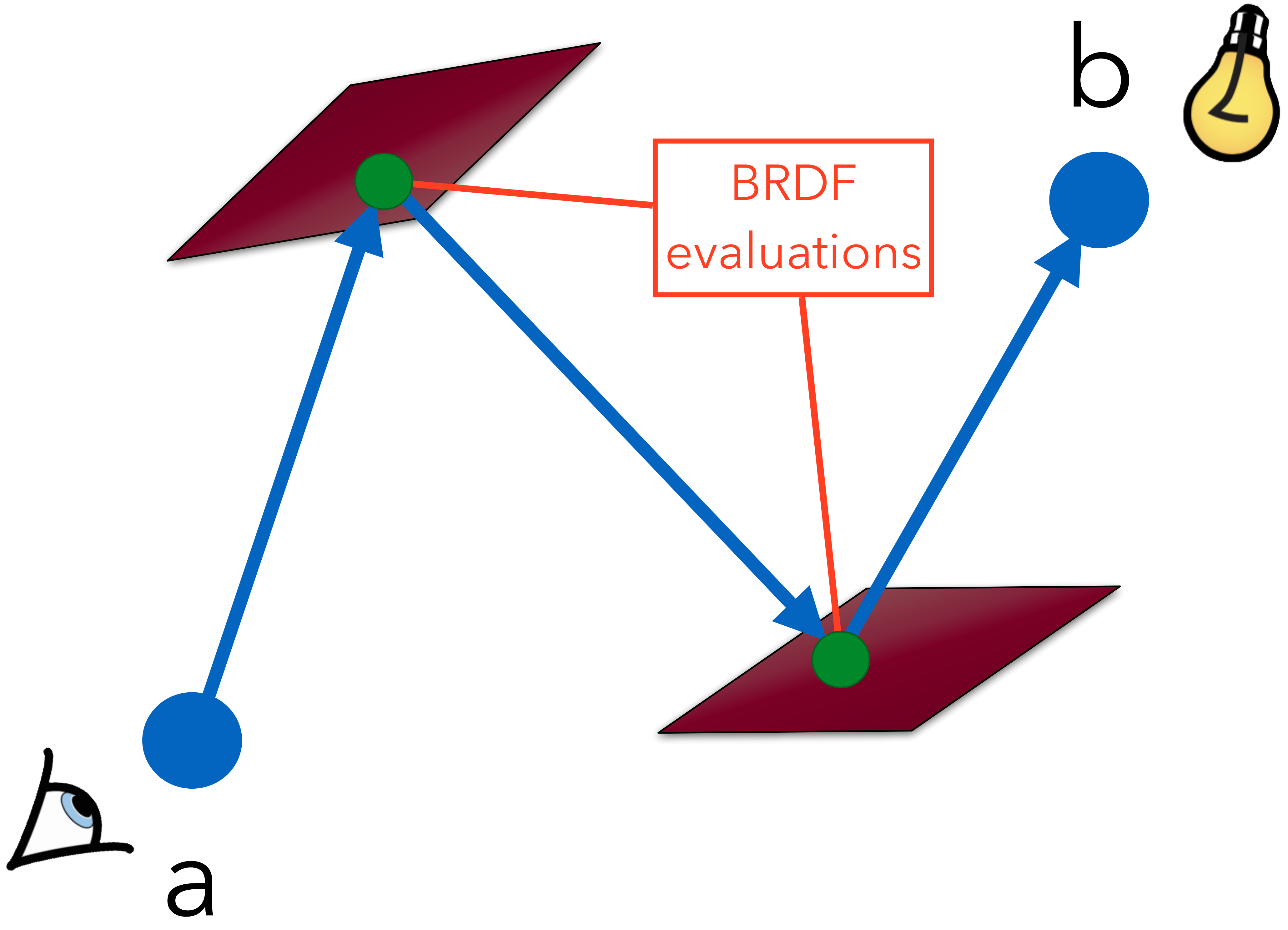
Volterra

$$T(a,b) = 1 + \int_a^b -\mu(x)\,T(x,b)\,\mathrm{d}x$$



Path Tracing (Fredholm)*

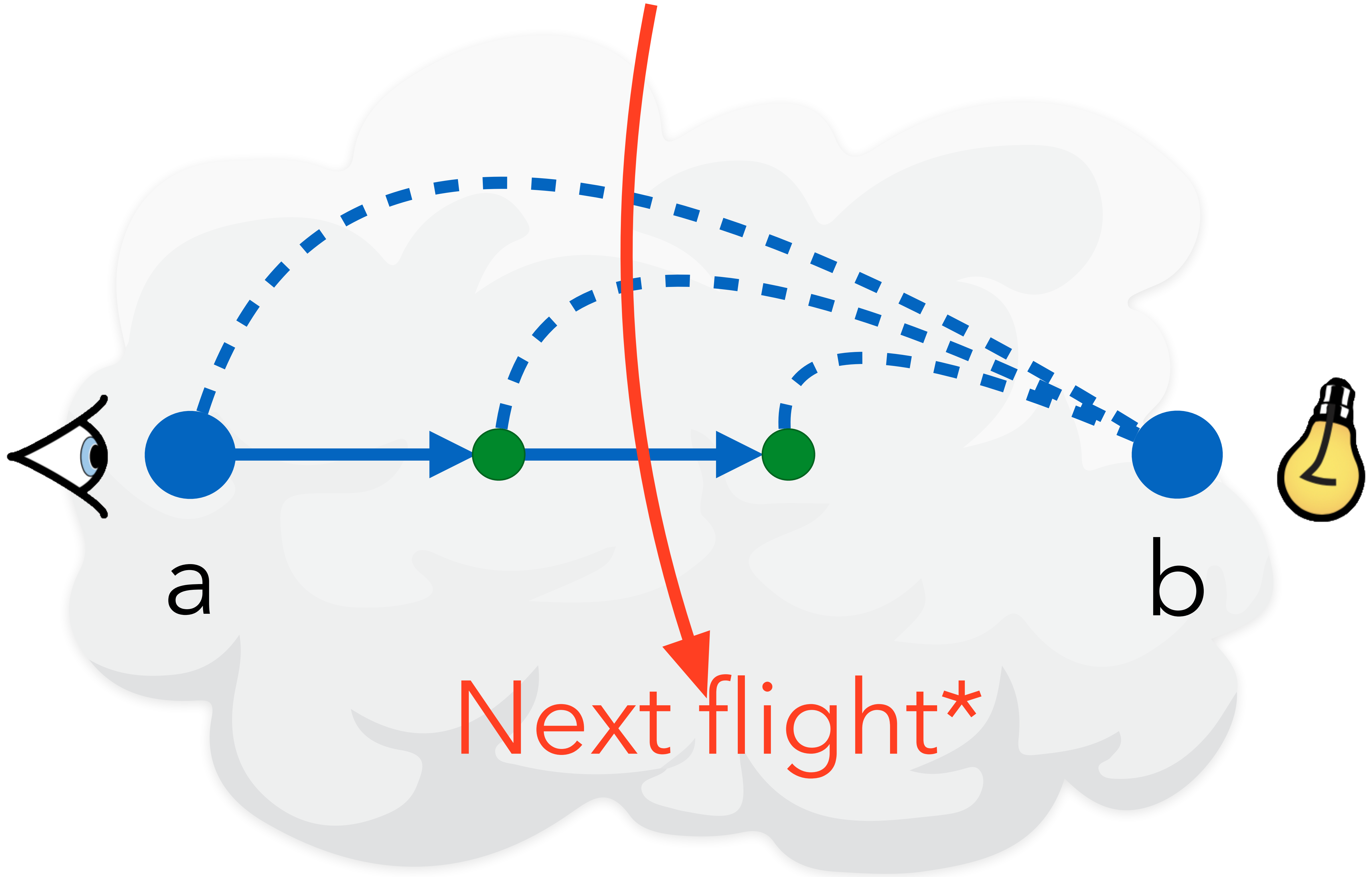
$$L(\omega) = L_e(\omega) + \int_\Omega f_r(\omega_j,\omega)\,L(\omega_j)\,\mathrm{d}\omega_j$$



*[Kajiya 1986]

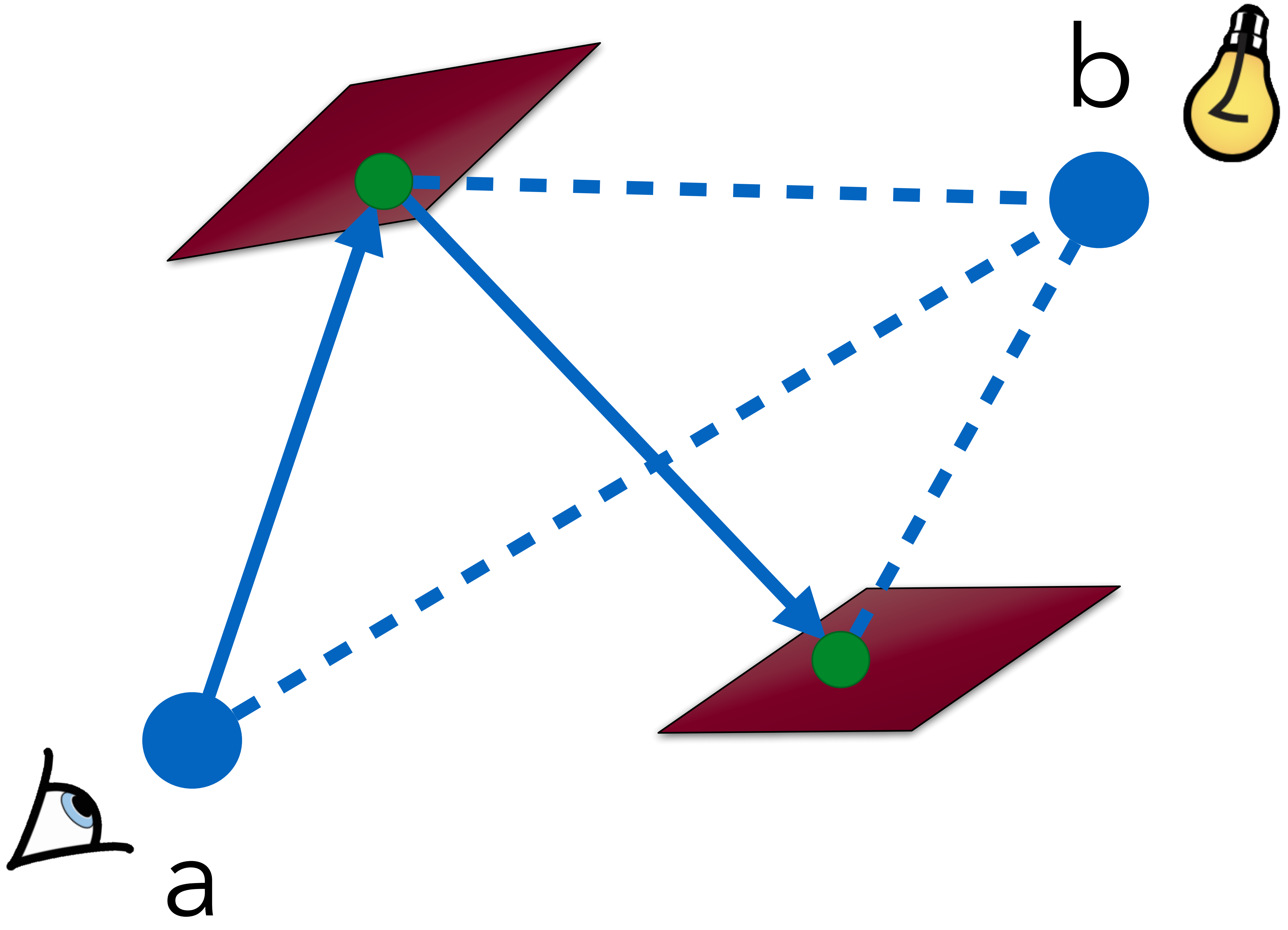
Better estimators?

Volterra



Path Tracing (Fredholm)

Path Tracing w/ NEE



*[Cramer 1978]

So far

- Volterra integral formulation
- Previous work
- Unidirectional path tracing analogy

Other integral formulations

Volterra

$$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$$

Rendering Equation

$$L(\omega) = L_e(\omega) + \int_\Omega f_r(\omega_j)\cos(\theta)L(\omega_j)\mathrm{d}\omega_j$$

Other integral formulations

Volterra

$$T(a,b) = 1 + \int_a^b -\mu(x) T(x,b) \mathrm{d}x$$

Rendering Equation

$$L(\omega) = L_e(\omega) + \int_\Omega f_r(\omega_j) \cos(\theta) L(\omega_j) \mathrm{d}\omega_j$$

Neumann Series*

$$I_j = \sum_{k=0}^{\infty} J^k L_e$$

*[Veach 1997]

Other integral formulations

Volterra

$$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$$

Rendering Equation

$$L(\omega) = L_e(\omega) + \int_\Omega f_r(\omega_j)\cos(\theta)L(\omega_j)\mathrm{d}\omega_j$$

Power-series

$$T(a,b) = \sum_{k=0}^{\infty}(-\tau)^k\frac{1}{k!}$$

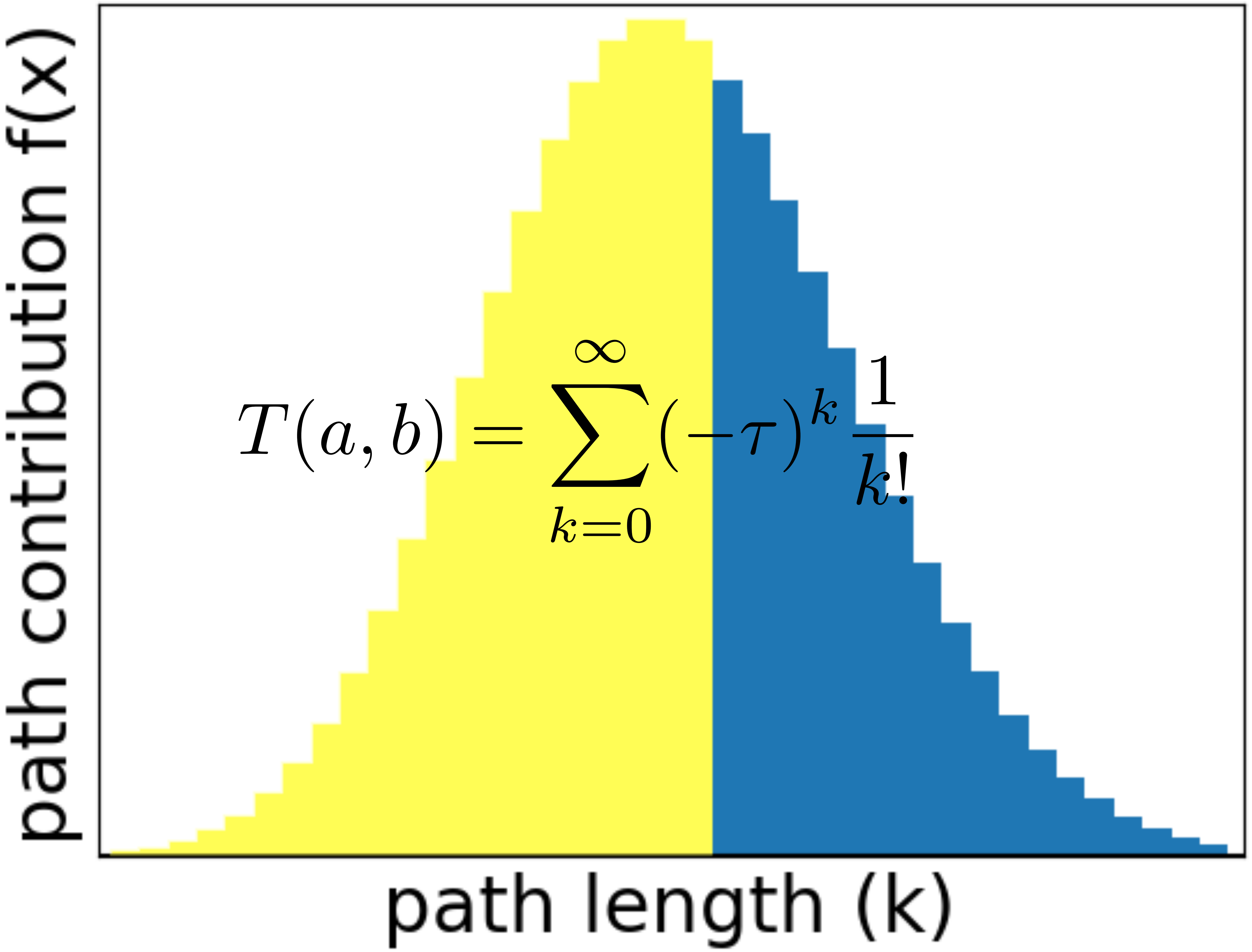
Neumann Series*

$$I_j = \sum_{k=0}^{\infty} J^k L_e$$

*[Veach 1997]

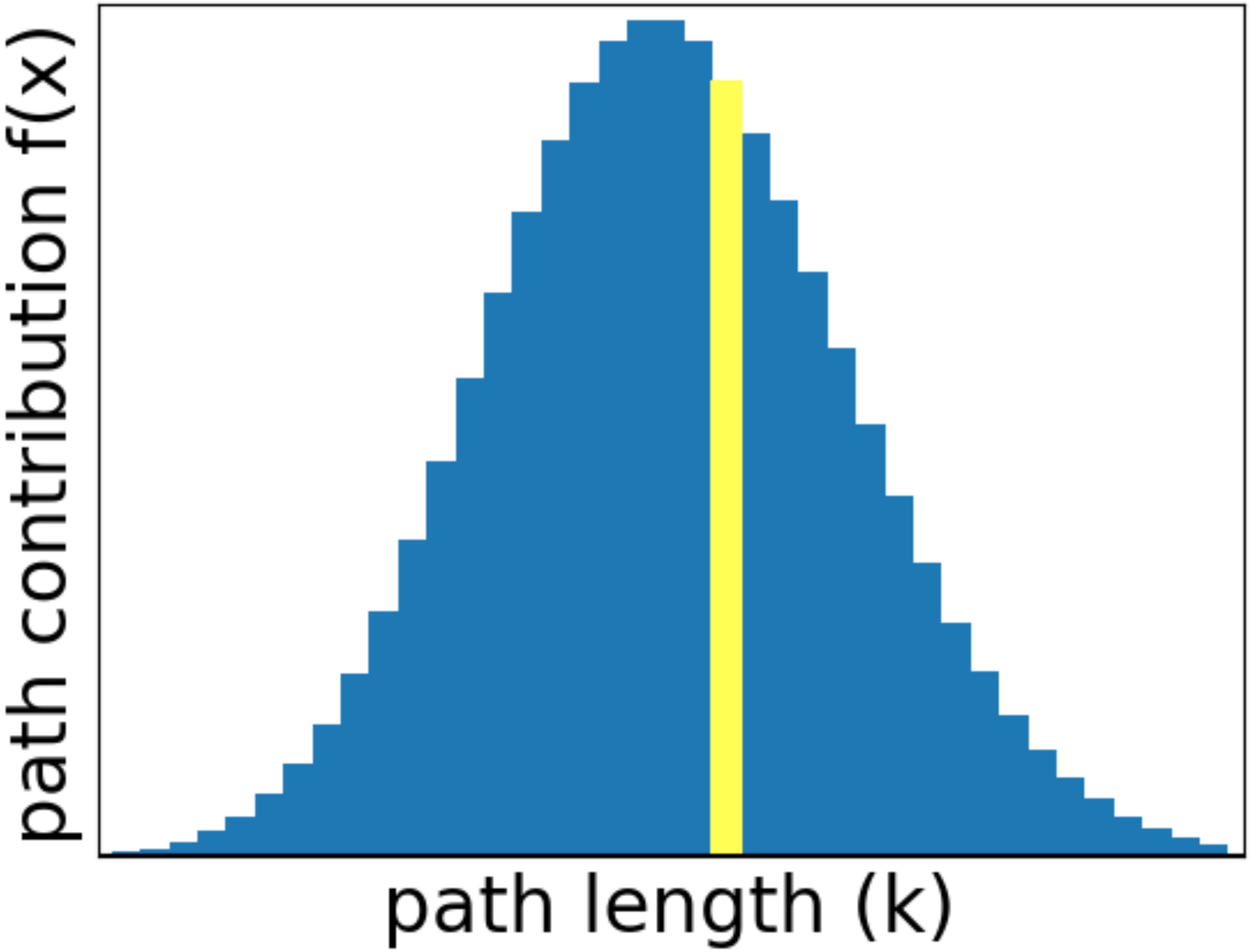
Power series formulation

Prefix-sum



$$T(a,b) = \sum_{k=0}^{\infty} (-\tau)^k \frac{1}{k!}$$

Single-term



Other integral formulations

Volterra	Rendering Equation
$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$	$L(\omega) = L_e(\omega) + \int_\Omega f_r(\omega_j)\cos(\theta)L(\omega_j)\mathrm{d}\omega_j$
Power-series	**Neumann Series**
$T(a,b) = \sum_{k=0}^{\infty}(-\tau)^k \frac{1}{k!}$	$I_j = \sum_{k=0}^{\infty} J^k L_e$
	Path Integral*
	$I_j = \int_\Omega f(x)\mathrm{d}\mu(x)$

*[Veach 1997]

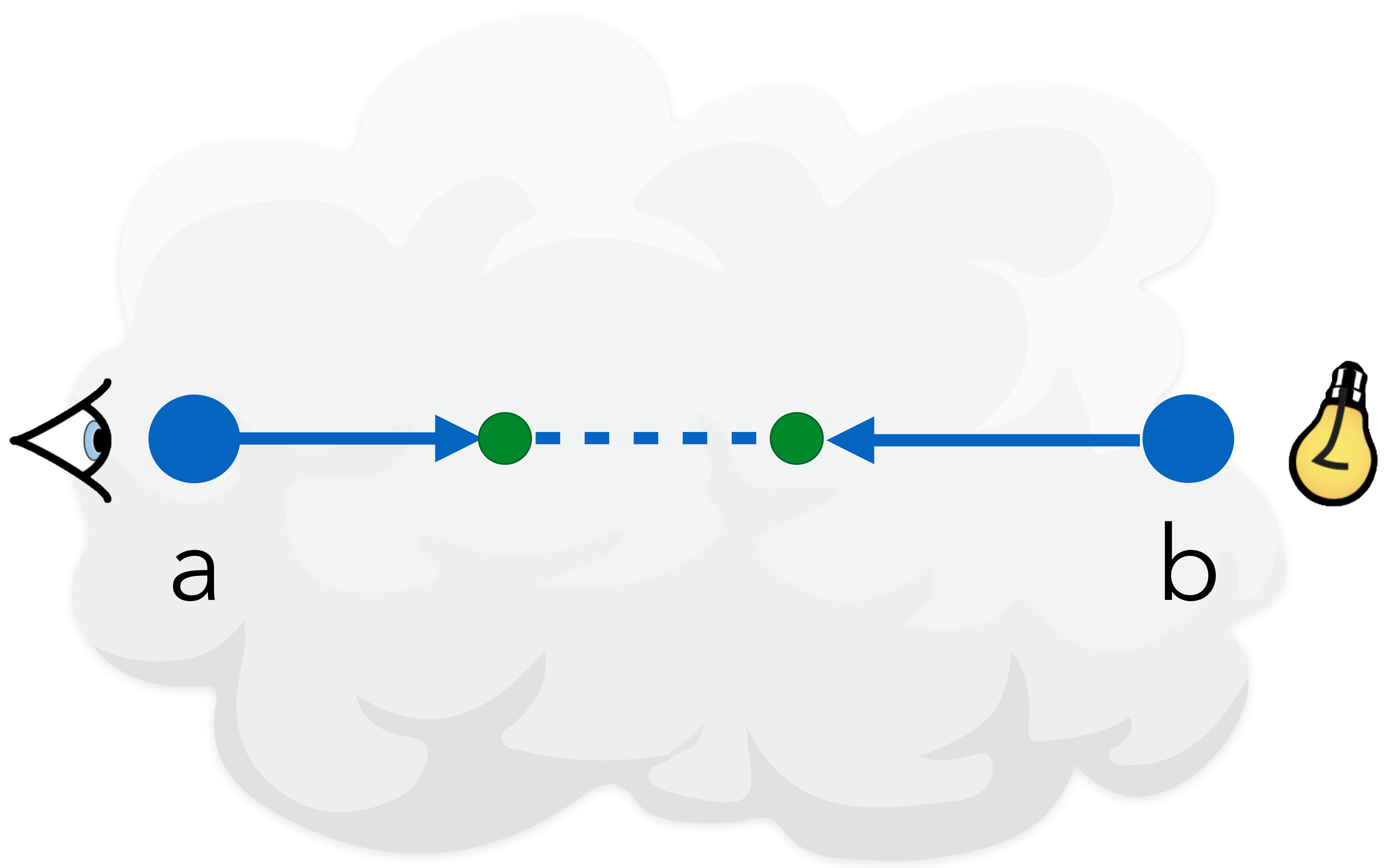
Other integral formulations

Volterra	Rendering Equation
$T(a,b) = 1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$	$L(\omega) = L_e(\omega) + \int_\Omega f_r(\omega_j)\cos(\theta)L(\omega_j)\mathrm{d}\omega_j$
Power-series	**Neumann Series**
$T(a,b) = \sum_{k=0}^{\infty}(-\tau)^k \frac{1}{k!}$	$I_j = \sum_{k=0}^{\infty} J^k L_e$
Hypercube	**Path Integral***
$T(a,b) = \int_{\mathcal{H}} f(\mathbf{x})\mathrm{d}\mathbf{x}$	$I_j = \int_\Omega f(x)\mathrm{d}\mu(x)$

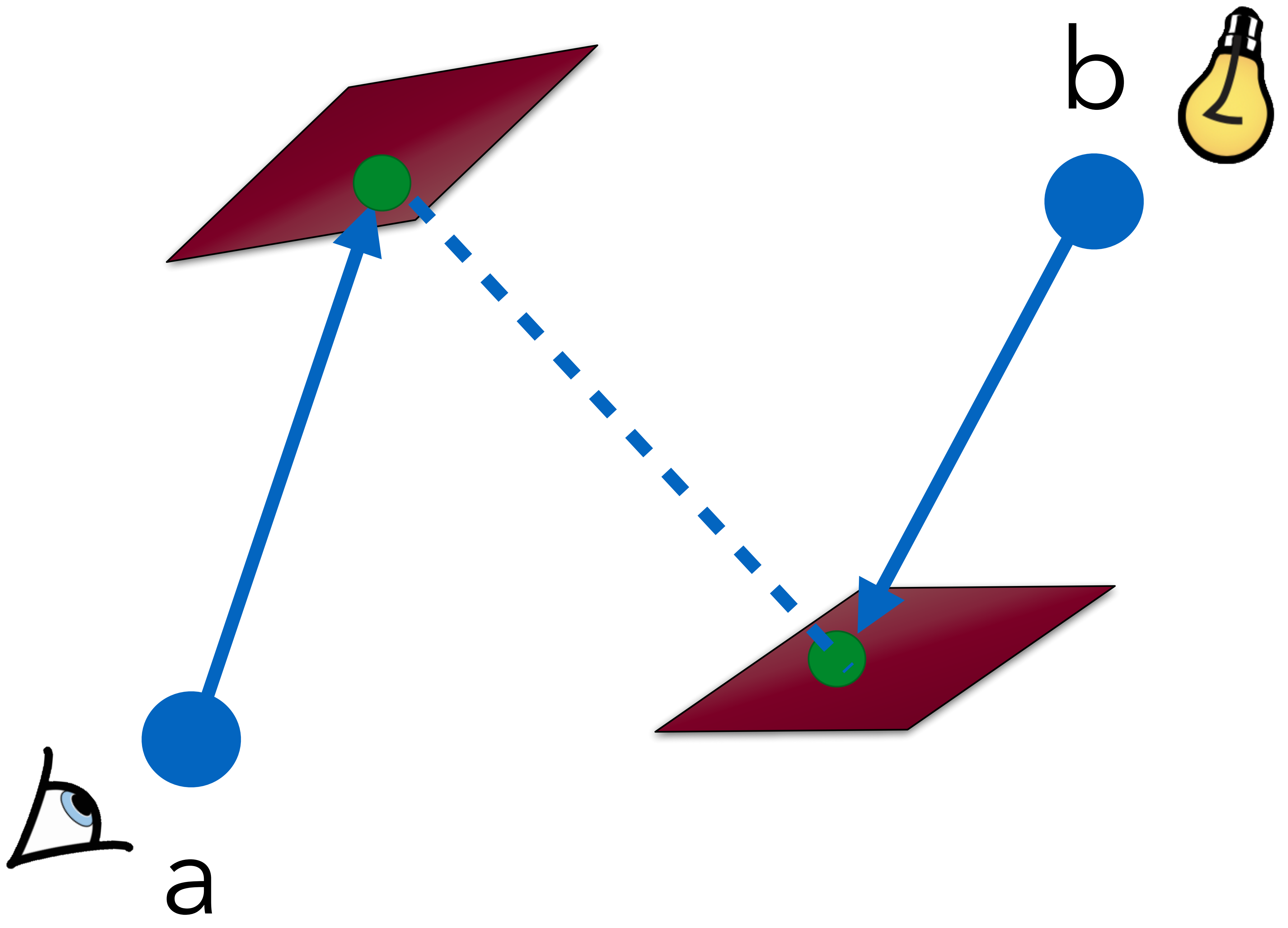
*[Veach 1997]

Hypercube integral formulation

Hypercube (bidirectional MIS)



Bidirectional path tracing*



*[Veach and Guibas 1994]

Estimator summary

Ours are in green

Volterra

$$1 + \int_a^b -\mu(x)T(x,b)\mathrm{d}x$$

Track-length
Ratio tracking
Next-flight

Power series

$$\sum_{k=0}^{\infty}(-\tau)^k\frac{1}{k!}$$

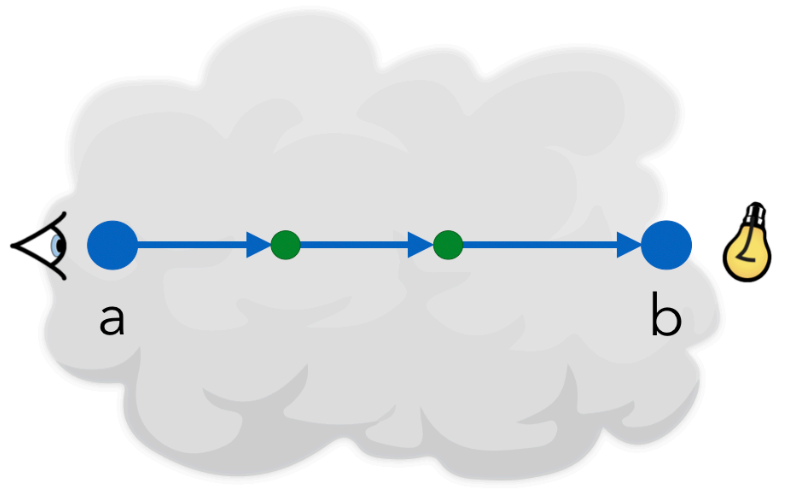
P-series ratio
P-series next-flight
P-series cumulative
P-series CMF

Hypercube

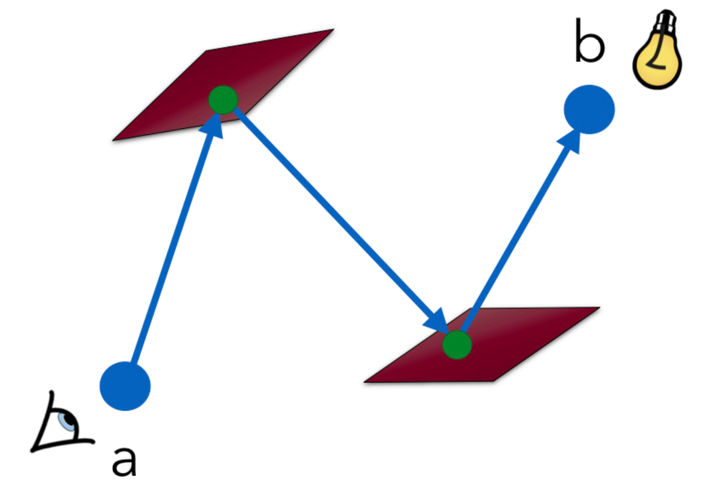
$$\int_{\mathcal{H}} f(\mathbf{x})\mathrm{d}\mathbf{x}$$

Unidirectional-MIS
Bidirectional-MIS

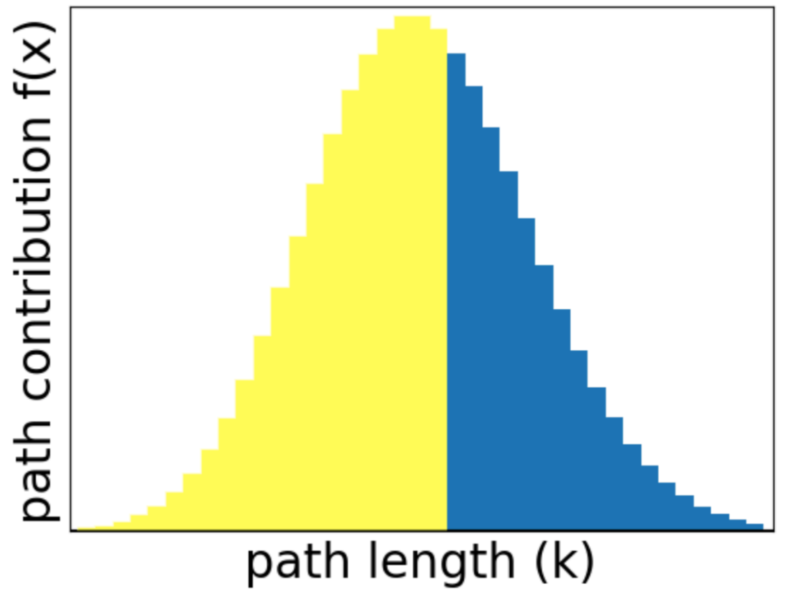
Volterra



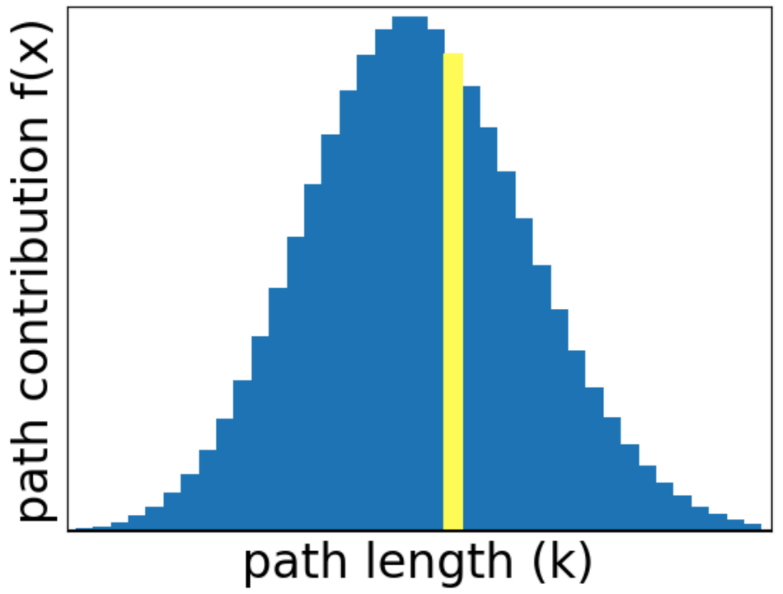
Path Tracing



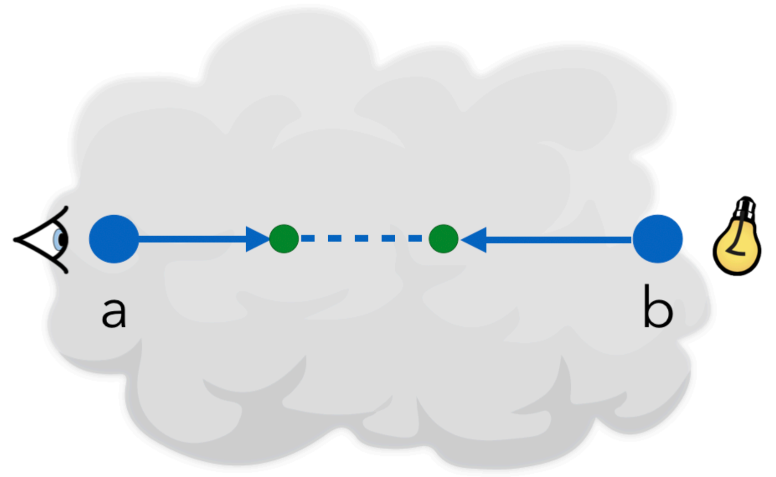
Prefix-sum



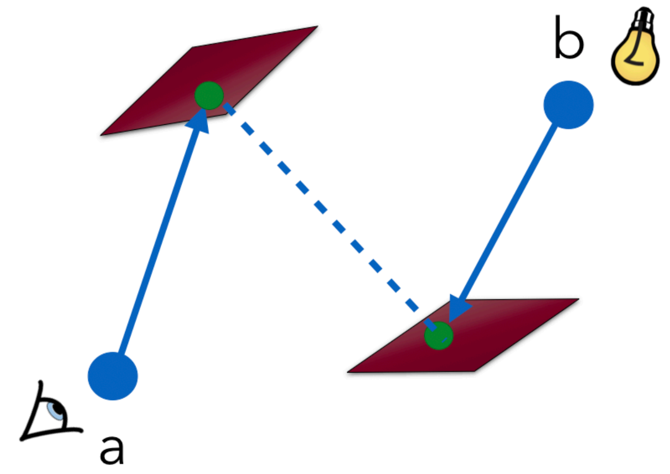
Single-term



Hypercube (bidirectional MIS)



Bidirectional path tracing



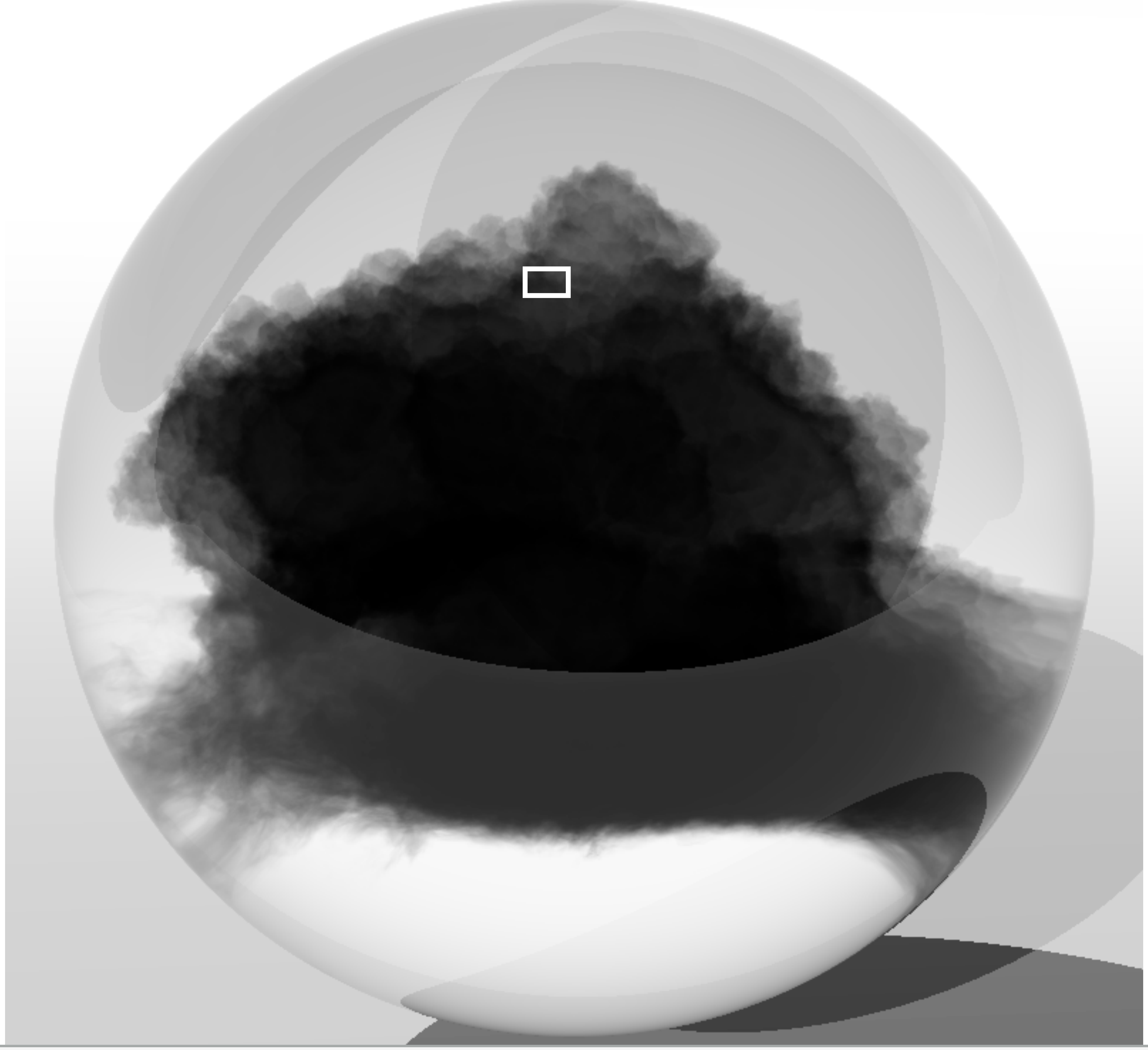
Results

Majorant / upper control

Tight upper control

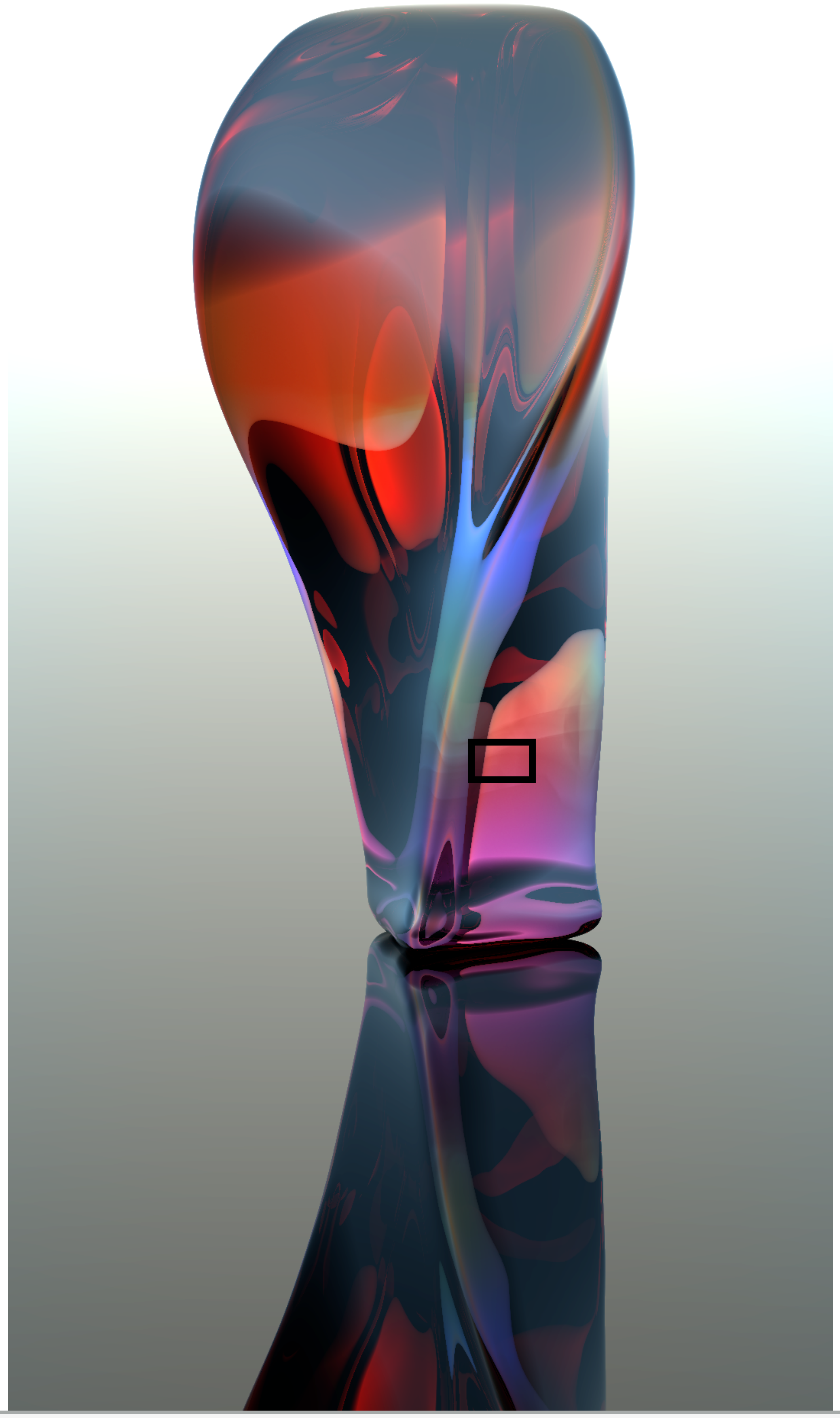

Ratio tracking
RMSE: 1.3e-2
Next-flight
RMSE: 3.6e-3
Reference
RMSE: 1.7e-6
P-CMF

Loose upper control



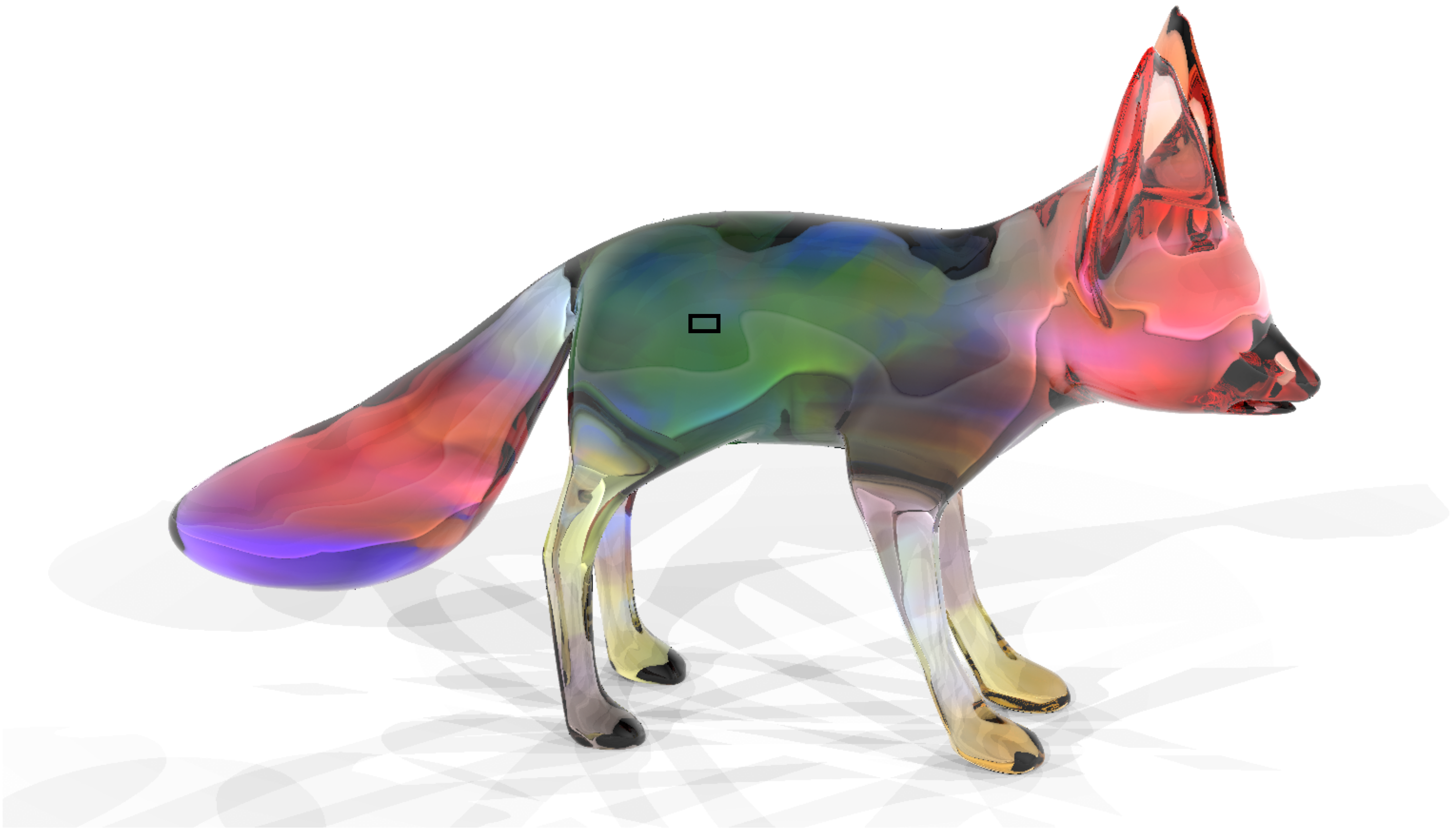

Ratio tracking
RMSE: 1.1e-2
Next-flight
RMSE: 1.0e-1
Reference
RMSE: 1.1e-2
P-CMF

Non-bounding upper controls



Ratio tracking (1x max)
RMSE: 1.0e-2
P-series Cumulative (1x max)
RMSE: 8.3e-3
RMSE: 2.7e-2
Ratio tracking (0.5x max)
RMSE: 8.6e-3
P-series Cumulative (0.5x max)

Stratification




Ratio tracking (independent)
RMSE: 1.4e-2
P-series ratio (independent)
RMSE: 1.4e-2
RMSE: 7.9e-3
Ratio tracking (stratified)
RMSE: 6.4e-3
P-series ratio (stratified)

Best estimator?

- There is no “best” estimator
- Loose controls - ratio
- Non-bounding controls - power series cumulative
- Other cases - power series CMF

Conclusion

- Integral formulation for transmittance
- Allows for flexible estimator design
- Re-formulated previous work
- 4 novel estimators

Future work

- Non-exponential media
- Application of the different integral formulations to sample scattering events
- More MIS estimators

Thank you!

Please visit: http://zackmisso.com

For the paper and supplemental

Code repository (GitHub):

ZackMisso/TransmittanceEstimation

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